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HUMAN DECISIONS IN COMPLEX SYSTEMS

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Part I. Survey of the Problems That Must Be Solved in Future Systems

HUMAN DECISIONS IN MISSILE SYSTEMS

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The development of weapon systems for the free world is an inescapable fact of the society in which we live today. The development of these systems in the future involves management system problems that must be solved if the required technical progress is to continue. It is the purpose of this paper to indicate the critical problems in communication between personnel and to suggest a form that the solution might take.

An essential distinction between "operational systems" and "development systems" must be made at this point. Operational systems are known by such names as Polaris, Thor, and Atlas, and are designed for operation by personnel of the armed services. Development systems are known by such names as Lockheed, Convair, and Douglas, and have as their task the provision of operational systems. To emphasize the system characteristics of the communication problems, development system is used in preference to the more familiar term management organization.

The primary source of these communications problems is the sharply increasing complexity of weapon systems. As shown in FIGURE 1, the trend in weapon systems is an accelerated growth in complexity. This complexity stems from the technical sophistication required to achieve planned objectives of a progressively more ambitious scope. An increase in the complexity of the operational system entails increased costs, lead time, and numbers of personnel on the part of the development system, that is, the management organization whose task it is to provide the operational system. FIGURE 2 shows the character of development system costs as they are affected by operational system complexity. Today's major systems are costly enough, although the United States and the Union of Soviet Socialist Republics are still able to finance them. If we fail to solve our critical problems, we may well expect that within 10 to 20 years further complexity will become prohibitively expensive.

The extent of the communications and, hence, the decision-making problem encountered in a development system even today, is well illustrated by the interfaces integral to a representative missile development organization with 100 departments. There are 4,950 interfaces resulting from the existence of 100 departments in such an organization.

FIGURE 3 graphically illustrates this point. If we define a department as a group of 10 or more individuals, we find that 100 departments are by no means unusual today. These 4,950 interfaces, it must be emphasized, are calculated on the basis of organizational units only, not as the number of people within each unit. The totality of interfaces actually experienced is a function, therefore, of the number of people in each department with interface responsibilities.

A significant concept arises in connection with these personnel interfaces and, furthermore, provides an understanding applicable to one solution that

can be foreseen for the communications problem. Of the thousands of people normally found in a development system of this scope, only a small proportion will contribute to significant decision making in weapon system design. By significant decision making is meant the selection, from several alternatives, of

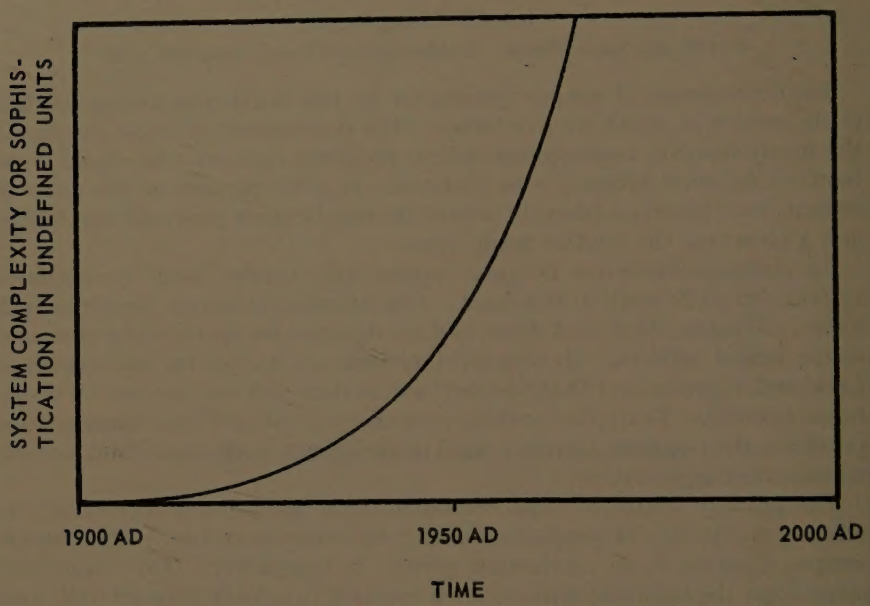

FIGURE 1. Growth in the complexity (or sophistication) of weapon systems with time.

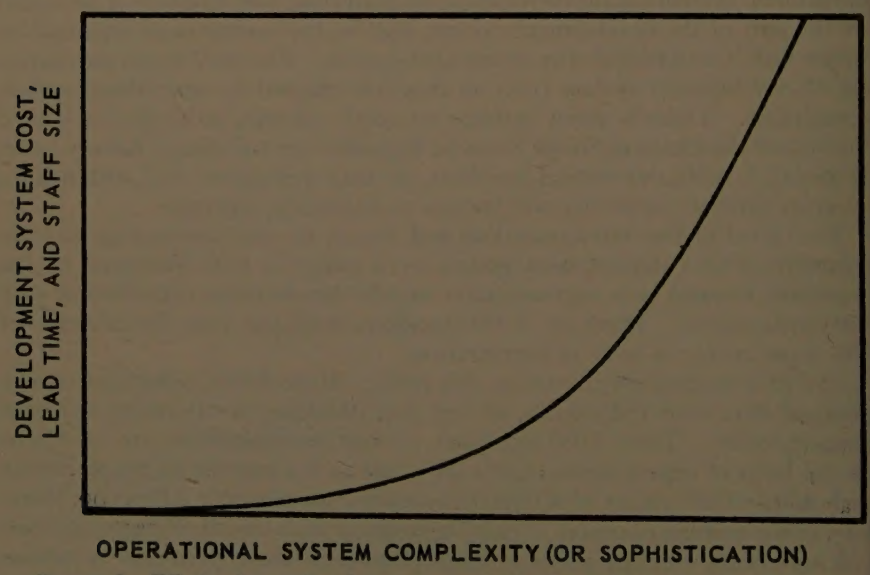

FIGURE 2. Effects of operational system complexity (or sophistication) on development system cost, lead time, and staff size.

a course of action having a direct effect upon the progress of development of the system. Examples of such decision making are: whether to use solid or liquid propulsion; whether to use manual or automatic checkout equipment; when to schedule the *n*th launch; what type of guidance system to employ; and what kind and number of people will be required to operate the system. Personal experience shows that less than 10 per cent of development system personnel are engaged in this type of significant decision making.

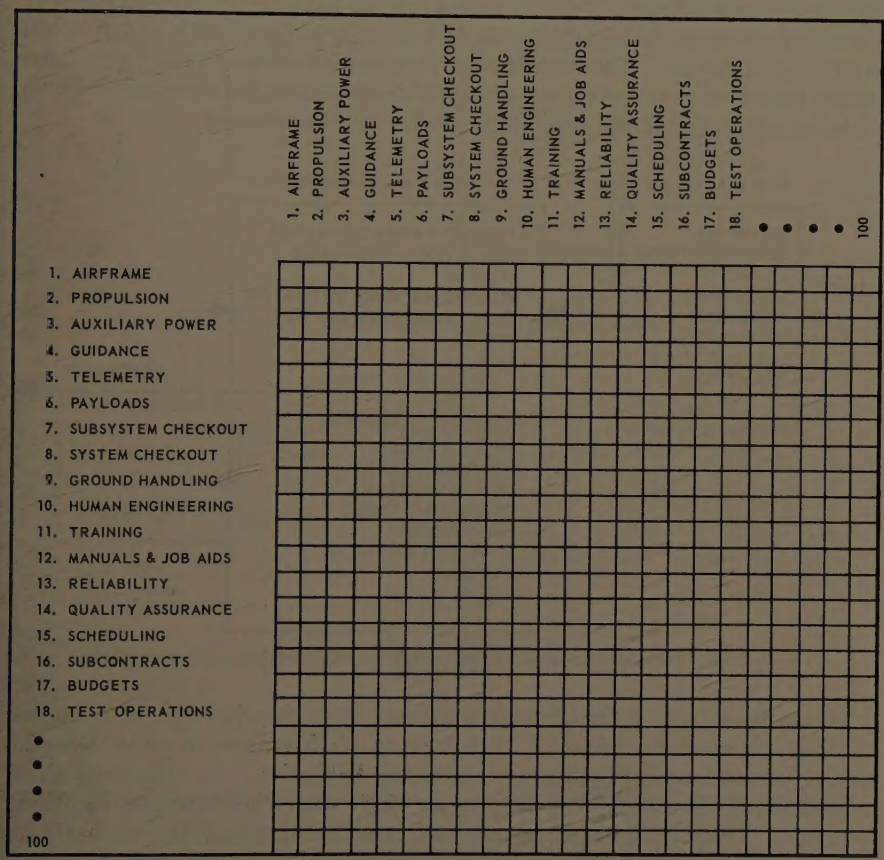

FIGURE 3. Interface matrix for 100 development system groups.

The remaining 90 per cent or more of development system personnel operate in accordance with detailed instructions, plans, prints, directives, and schedules to produce a product meeting well-defined criteria of excellence. Both the prototype model and the criteria of excellence are supplied from sources outside the "typical" worker in a development system. Personnel in this "typical" category include turret lathe operators, drill press operators, platers, milling machine operators, electronic assembly men, typists, and file clerks. A decision model for such personnel is offered in FIGURE 4, which shows the development of their product in conformity with the program governing their work.

It may be seen, therefore, that essentially no critical decision-making problems exist for individuals falling into this category of development system personnel.

For the personnel manning well-designed operational systems, even fewer decision-making problems exist. High reliability of the personnel aspects of an operational system requires that essentially all decisions be made in advance for these personnel. The decision to employ the operational system is, of course, made outside the missile system itself by a chief of state or some other duly authorized authority. Once such a system has had its "button pushed," a series of events occur in mechanistic fashion in accordance with a plan known as the "countdown." The performance of personnel varies according to distribution functions that have their corresponding means, variances, and limits. Performance, therefore, is predictable in the statistical sense.

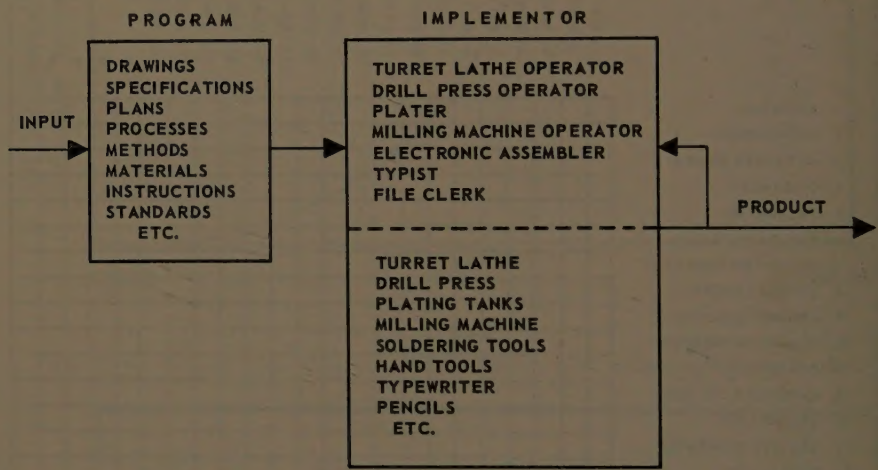

FIGURE 4. Decision model for most missile system development personnel.

The typical task of operational personnel is to discriminate between the go/no-go alternatives displayed at their work stations, and to undertake previously-planned action appropriate to each alternative. In this sense these individuals do not *decide* anything other than what the displayed condition portends. Their equipment, instructions, and training provide the basis for this interpretive decision and for their consequent action. Indeed, an apparently high-level decision by the launch director to scrub vehicle No. 12 and substitute vehicle No. 14 is, and should be, the result of (1) recognition that vehicle No. 12 meets the no-go criteria, and (2) knowledge that the plan specifies substitution of vehicle No. 14 when and if No. 12 is scrubbed. FIGURE 5 is offered as a model for the functioning of operational missile system launch crews. These last two decision models were conceived with the help of insights developed by L. J. Fogel in his paper in this monograph.

By accepting the concept of the creative minority we can delimit the communications and decision-making problem, for we have seen that operational system personnel and most development system personnel may be excluded

from the ranks of significant decision makers. It is for the minority that improved communication techniques must be found, as shown by the definition of the significance of their decision making.

The means of communications available today for this creative minority are primitive. These personnel communicate by writing on paper, talking face-to-face, or speaking over telephones. Except for the lack of telephones, the ancient Egyptian had comparable facilities.

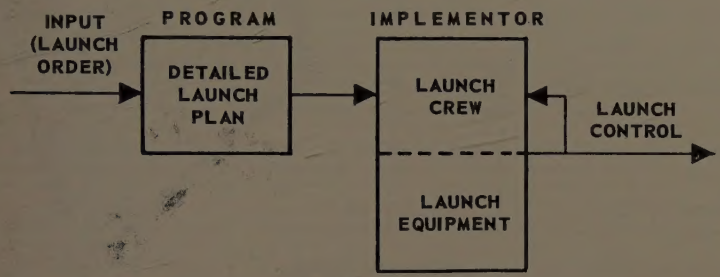

FIGURE 5. Decision model for operational missile system launch crew.

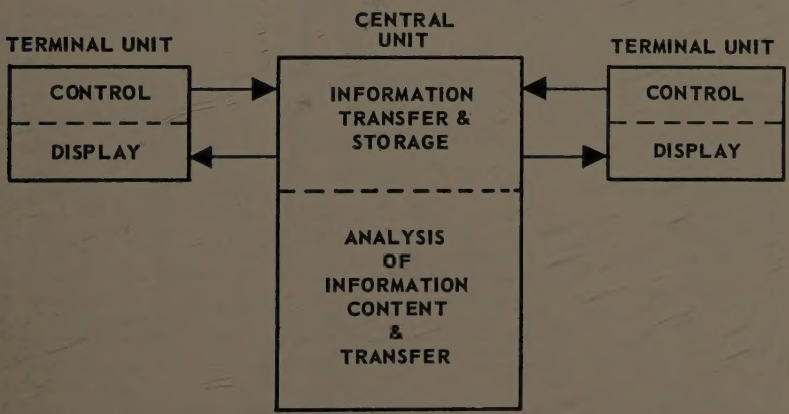

FIGURE 6. Block diagram of information system for development system personnel.

It is patently impossible to use the spoken word to handle all essential communications, and paper is therefore the only remaining alternative. However it is also impossible for every minority member to send copies of every decision to every other minority member, or even to every other department head. The current practice is therefore to exercise "judgment" and to send copies to individuals who "need" them or "ought" to have them. In systems with nearly five thousand interfaces it is inevitable that such judgment is not infallible.

The result of these communication system constraints is that today's minority must spend a phenomenal amount of time waiting for information, tracking the source of specific information, developing "informal" information channels,

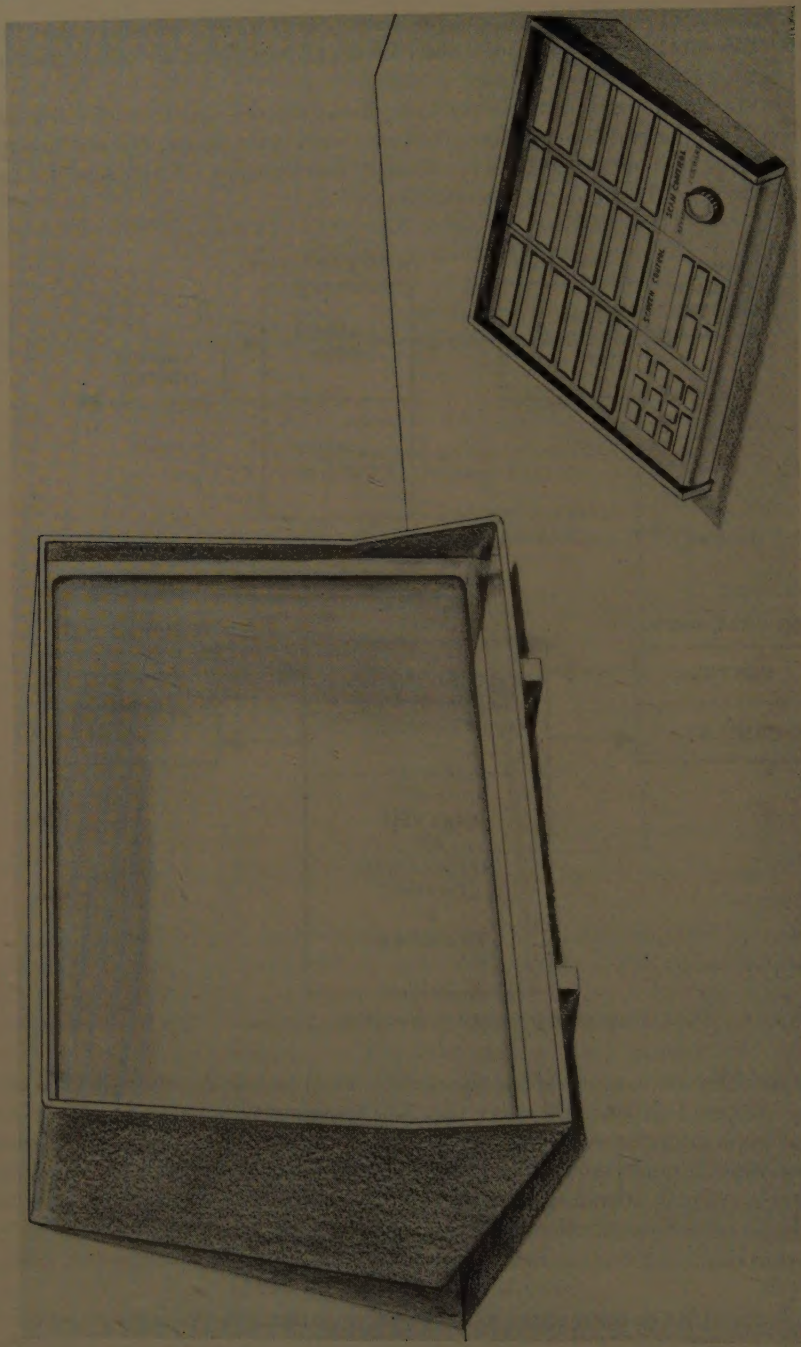

FIGURE 7. Close-up view of information system terminal unit (artist's concept).

and coordinating information. Days may pass until a requested document is finally obtained, and then it may prove inapplicable to the problem at hand. Such examples might be cited at great length; the blame is usually placed on the "system," as indeed it should be.

Management, for its part, constrained by today's primitive communication tools, has but four means at its disposal to improve development system operation. These are (1) to issue directives, (2) to revamp organization charts, (3) to reassign personnel, and (4) to add more personnel. The apparently be-

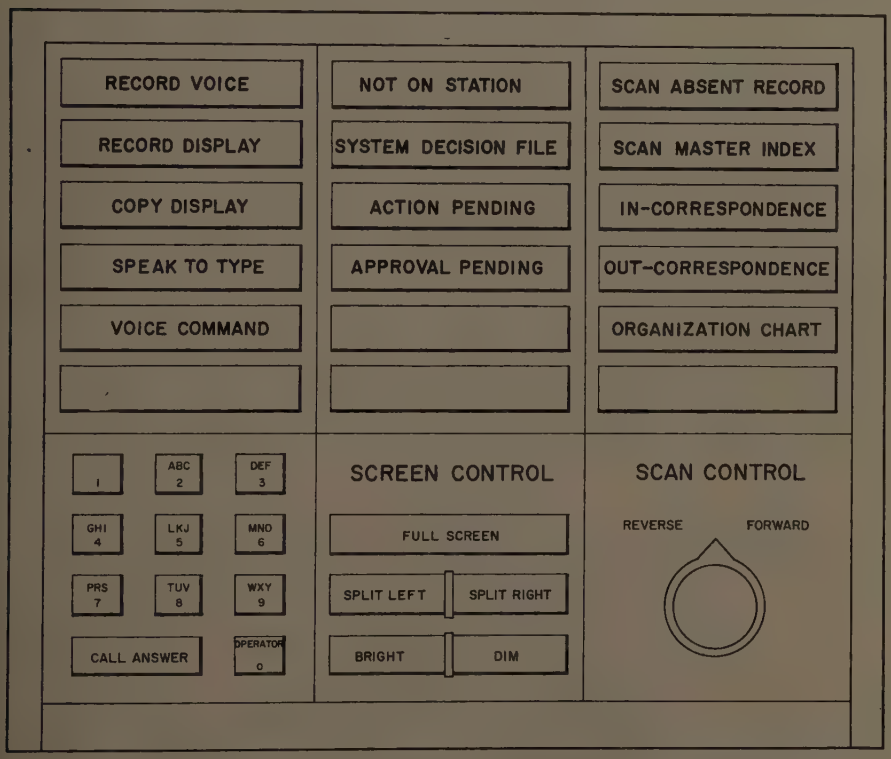

FIGURE 8. Control panel for information system terminal unit (artist's concept).

wildering series of reorganizations, personnel reassignments, new directives, and new personnel associated with modern weapon system programs should be viewed as the exercise of the only alternatives available to today's management.

A means must be found, therefore, to relieve the creative minority of an increasing burden resulting from the sophistication of new systems. Unless such means are found, we shall be reducing the rate of contribution of this group even as we ask for increased performance. Solution of this problem for the creative minority may be realized by providing easier access to information required for significant decision making, faster flow of information between personnel, and improved methods of information display.

One step that might be taken to effect these changes is the introduction of "Information System Equipment," based on the assumption that all physical exchange of paper as a medium of communication can and should be eliminated

TABLE 1

SOME FUNCTIONAL CAPABILITIES OF INFORMATION SYSTEM TERMINAL UNIT

Control options
- (*a*) Request specific technical information.
- (*b*) Request specific administrative information.
- (*c*) Request delayed playback of message originating from this unit.
- (*d*) Enable access to the display(s) of this unit from other terminal unit(s).
- (*e*) Request delayed playback of message(s) from other terminal unit(s).
- (*f*) Record or copy incoming/outgoing messages.
- (*g*) Enter data into information system.
- (*h*) Translate voice message to typed copy.
- (*i*) Search selected listings (incoming or outgoing correspondence, specific catalogue sets, etc.)
- (*j*) Select desired document, correspondence.
- (*k*) Control speed of advance or retrace through document.
- (*l*) Notify information system that terminal unit will be untended.
- (*m*) Dial other desired terminal unit(s).

Display options
- (*a*) Technical information.
- (*b*) Administrative information.
- (*c*) Real-time or delayed playback display of message(s) originating from this unit.
- (*d*) Real-time or delayed playback display of message(s) from other terminal unit(s).
- (*e*) Display of information originating from central unit ("tickler file," estimated "best" time(s) of access to desired other terminal unit(s), etc.).
- (*f*) Display of catalogues, listings, or other documents in storage in central unit.

TABLE 2

EXAMPLES OF ANALYSES OF WHICH CENTRAL UNIT IS CAPABLE

Delay time analysis for system personnel
- Time to return "telephone" call
- Time to indicate approval
- Time to acknowledge receipt of information
- Time to obtain information
- Time to complete assigned task

Link analysis
- Who is linked with whom in actual practice
- Link strength
- Link frequency
- Qualitative characteristics of the link
- Sequential dependency characteristics

Decision-making analysis
- Decision source
- Decision adequacy

for minority personnel. It is desirable for present purposes to consider none of today's restrictions of costs or feasibility. A block diagram of this system is shown in FIGURE 6 and consists of two basic components: an Information System Terminal Unit (or simply "Terminal Unit"), and an Information System Central Analysis, Storage, and Monitoring Unit (or simply "Central Unit"). The Terminal Unit serves two primary purposes: communication between people and information retrieval. The Central Unit serves three

primary purposes: analysis of development system information and operations, information storage, and system monitoring.

The Terminal Units are available in Executive and Regular models, and a close view of the Regular model (FIGURE 7) shows a display screen and control panel. An artist's conception of some of the elements of the control panel is shown in FIGURE 8. Basically, the purpose the Terminal Unit is to allow an individual to summon to the screen any system information, either by voice instruction or by button-pushing and, correspondingly, the insertion of information into the system and/or communication with other members of the system. A more complete list of functions the Terminal Unit is capable of implementing is shown in TABLE 1.

The Central Unit receives inputs from Terminal Unit positions, and these inputs are tagged according to date, time, originator, addressee, and subject, and put into storage for subsequent retrieval, analysis, and monitoring. Access to many of the analyses, as illustrated in TABLE 2, would be limited to higher management. Such analyses, based on a quantity and quality of data unavailable from today's systems, would assist in implementing the customary managerial functions.

In summary, we have seen that critical decision making problems exist for neither operational system personnel nor for the majority of development system personnel. A critical problem, however, does exist for the creative minority element in development systems, and relief of the situation by providing easier and more rapid communications, perhaps along the lines suggested, is mandatory if future objectives are to be realized.

Acknowledgments

I acknowledge with appreciation art and industrial design work by D. H. Richardson, technical suggestions by R. C. Smader, editorial advice by C. W. Parker, and secretarial assistance by Anne Cox.

HUMAN DECISIONS IN COMMAND CONTROL CENTERS

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Introduction

Men have been designing command control systems for as long as they have found it necessary to combine for the accomplishment of almost any mission, military or otherwise. As soon as a structure was established for the giving and taking of orders and for the transmission of information in a more or less systematic way between members of even the most rudimentary organization, the essentials of a command control system had emerged.

As long as the tasks to be performed remained relatively uncomplicated, the time available for their completion relatively great, the risks associated with failure relatively low, and the size of the group relatively small, the formal relations among the elements of the organization in question could remain comparatively few and simple. Technological advances have made the missions and tasks more complex, but they have concurrently provided the means for operating under more complex rules in a more demanding world. While it is necessary to recognize the capability provided by modern techniques, it is important to avoid being misled by the complexity and magnitude of automated and semiautomated systems into regarding the problems as entirely new. Basically, they are elaborations of former problems, newly emphasized by the new environment.

The modern problem of command is distinguished from earlier ones by the rate at which information must be handled and decisions made. This distinction may be clarified by dividing the problem into its two readily observable aspects: (1) complexity and (2) urgency. (While the ensuing discussion describes the military case, analogies in the commercial, industrial, and governmental fields are not hard to find.)

(1) That a given decision both involves more data and implies more consequences than it used to follows from the increased scope of the "battlefield," the greater range and effectiveness of weapons, the greater variety of alternative actions available, the larger amount of mobility both possible and desirable, the increased interdependence among line, staff, and support activities, and improved methods of prediction and planning.

(2) The increased urgency for proper decisions results from the reduced time required for weapon delivery, the magnitude of available destructive effect, the irreversibility of commitments, and the possible suddenness of onset of the conflict state.

Command decisions depend (hopefully) upon data either currently supplied or stored. This paper attempts to describe the classes of decision required in command, which may be thought of as the process of achieving desired objectives through selective commitment of available resources.

Command *centers* are typically nodes in networks constituting command control *systems*. The notion of a "center" must not be misinterpreted as implying centralization of decision. It simply indicates a place where some

decisions are made. Some centers are simple, consisting of a single decision element. Others are complex, containing elaborate networks.

Definition of system. A system is to be thought of for present purposes as a collection of elements organized to perform a set of functions, during a specifiable era, in an environment known only uncertainly. "Element" refers to a more or less inseparable collection of properties at any of a number of levels of discourse. In the present context men and machines are considered elements. The characteristics of systems that deserve to be stressed are the organization of the elements, the uncertainty under which they characteristically operate, and the fact that there is a purpose for which they are intended. In general, systems are embedded in larger systems and embed smaller ones.

Classes of Decision

A decision will be regarded as a selection from among alternatives. An action is a commitment of resources. An action decision is therefore a selection from among a set of alternative resource commitments. The resources available for commitment by a commander include men, money, material (including weapons), energy, information, and time. In the following it is not to be inferred that decisions are necessarily made in the order described. On the contrary, the most characteristic operation of command centers involves continual and concurrent decisions of all kinds.

State of the world. Under the assumption that a good decision is an informed decision, it is at least desirable for the decider to be aware of the present state of the world. This state is described by the values of a set of relevant variables that include not only absolute values but indications of departure from expected, desired, or required values. The question of how these expectations are built up is an extremely interesting one that is, of course, receiving considerable scrutiny. Since a complete description of the world can be neither achieved nor used, the decider must operate upon information in terms of classes that permit orderly storage and utilization. The selection of meaningful classifications is one of the challenging decision problems of command systems.

Inasmuch as the decision process is extended in time, the decider must sense the change in relevant variables over some period of the recent past, the length of which depends on each particular piece of information involved. This is important for two reasons. First, any given change may be a trigger to action. Second, for sampling purposes the decider is forced to estimate what may be called the relative stationarity of the process being observed. For if he is to generate estimates based upon statistics he must determine an appropriate sampling interval or sample size.

Since estimates of the present state are used to contribute to action selection and since initiation of action generally takes time, it is important for the decider to predict changes that are likely in the future. Of course, unpredictability grows as a function of time. Moreover, the future state is partly a function of the action vectors chosen by the decider, his antagonist, or others.

The military decider needs information on actually or potentially hostile agents' capabilities, limitations, disposition of resources, policy, and other special characteristics. He must take account of the relevant neutral environ-

ment (physical, economic, or other), and he must be aware of his own strength and limitations, disposition of his resources, policy, and the special characteristics of his subordinate commanders and his staff. He needs to know the present state internal to the center, at other centers, and in other systems with which his system interacts. The problems of acquiring information of these three kinds are quite different, since the sources are polarized negatively, neutrally, and positively.

Since the decider's knowledge of the world is always imperfect, his descriptions must be partly incomplete, partly statistical in nature. Thus he is interested in knowing on which variables he has information and on which variables he is lacking information, and he must estimate the probability distributions associated with all stochastic variables relevant to his problem. It is in connection with this last kind of knowledge that information on capabilities, limitations, interest, preferences, and experiences of his opponents is valuable, for they shift his estimates of probabilities.

Available alternatives. At any given time the decider must analyze what actions are available to him. The set of available actions is a function of time. Some changes may be quite sudden, due to such factors as enemy action, political activities, and technological breakthroughs; or they may be gradual, due to obsolescence, price changes, and the like. In any real-time situation, the various aspects of complex action unfold at different rates, so that the consequent constraining effects vary progressively. Moreover, the actions available at a given time clearly depend upon the decisions made up to that time. In this situation, the commander must decide what new alternatives have been introduced and what old ones eliminated. It is important to note that obtaining additional information about any or all of the categories of interest is one kind of action, and that the alternatives effectively available depend upon the state of knowledge of the decider.

The decider is interested not only in his own alternatives but in those of the competitors, opponents, or enemies with whose alternative actions he will have to cope and which he, therefore, must predict; he also needs information on the decisions of others who may affect the processes, either as catalysts or as determiners of boundary conditions or else by becoming participants. Since information is expensive, the measures required for its acquisition deprive a commander of degrees of freedom just as effectively as any other resource commitment. One of the problems in training deciders is in convincing them that information is a commodity for which it is worthwhile to sacrifice a certain amount of their operational capability.

Predicted outcomes. The possible outcomes of a selected action diverge in time, as do the branches of a tree in space. Even during a limited interval the possible consequences of an action may be too numerous to list, and they may continue to unfold indefinitely and to interact with other factors. Thus, as a basis for selecting actions, the predicted outcome is a source of considerable uncertainty. Nevertheless, without a modicum of predictability, purposive behavior is impossible.

The relevant aspects of the consequences of an action include the expected "value" as a function of time; the new state of the world, expressible stochas-

tically; and a new set of available alternatives (which may be subsumed under the description of the world but is better distinguished for present purposes).

The outcomes of decisions are functions of a set of relevant variables of which the identities or distributions may be partially unknown. At the present state of the art of war, management, or government we do not know all the relevant variables, nor are we able characteristically to specify, for those we do know, the distribution functions as a function of time. To gain some insight into the possible outcomes of complicated processes similar to those we expect to encounter in practice, such activities as war games, or business games, have become fashionable as part of the process of planning. In using these games, explicit or implicit decisions are made about the applicability of the outcomes of the games as estimators of the outcomes of actual operations, and about the degree to which the game simulates the operational situation.

Objectives and criteria. The operation of command control systems is purposeful; that is, it has objectives. This implies that a set of decision criteria are being applied, whether or not they can be identified, but this set is not necessarily the same at all times for a given system.

Here again the military commander may be taken as an interesting example. After a particular kind of attack his criterion or ordering process on the set of available action alternatives may be very different from that he held before the attack. Moreover, it is not at all clear that this criterion could have been anticipated in all cases. It will depend, among other things, upon his estimate of the desires, criteria, or ordering processes of others besides himself, such as his commander and his commander's commander.

Moreover, and this point is interesting to those who feel that the place of human beings in future systems is tenuous at best, objectives and criteria are not necessarily computable functions. If there can be said to be a field for technological improvement of human beings in connection with command control centers, the direction for such improvement seems to be in what is wanted, how it is wanted, and how the want is formulated, so that it can be realized by the assemblage of men and machines that constitute the system.

Conclusions

The foregoing discussion gives only an incomplete idea of the complexity of the decision problems faced at the nodes in a command control system. Since information-gathering and information-keeping resources must always be limited, it is not at all evident that at any given time all relevant information about the state of the world is or can be made available. We need to know what constitutes an "acceptable" set of information. Nor is it obvious that a "best" decision can be associated in advance with each discriminable state of the world, so that when the one occurs the other can be automatically summoned. It is not even clear that a satisfactory description can be given of the likely outcomes of each of the available alternatives at any instant. Even the objectives, on the basis of which preferred subsets of available alternatives could be and are formed, are subject to progressive development, not necessarily describable in advance. Moreover, these four kinds of decisions interact vigorously.

The wonder is that decisions better than chance seem to occur. The paradox is that I seem to think I can tell what *is* better than chance.

However, decisions *are* made. Objectives *can* be stated, and they do remain valid for a while. The state of the world *can* be assessed, and the assessment used selectively upon what is at least an implicit set of alternatives. Resources *are* committed, and organizational changes *are* made. These things are possible because rates of change in the processes to which we are required to respond tend to be bounded. The military problem has always been to deal with events, actual or possible, in which these bounds are exceeded. The hunger for both information and the means to process it effectively and rapidly has grown acute in proportion to the magnitude of the change that may be accomplished abruptly.

By virtue of present methods of swift delivery of high yield weapons, we may be approaching the limit of increase of rate of change. The command control problem may, in certain cases, have become insuperable. In the face of bilateral inability to insure adequate decisions, potential antagonists may eventually abandon cataclysmic conflict for subtle conflict, the hot for the cool war. In subtle conflict, the objective will be to obscure rates of change of relevant cues about one's own actions while detecting and countering those of one's antagonist. To protect these indicators they should be kept below a certain attention threshold. This threshold can be kept high by creating a series of incidents specifically designed to be obvious and, at the same time, incoherent with respect to acts that are instrumental in terms of planned objectives. While required response speeds may then decrease, the information requirements for decision will increase to a point where it is unlikely that any net decrease in processing rate will be possible. In fact, as our sophistication in the use of information increases, it is likely that required rates will continue their upward climb.

It is interesting to note that, should the trend from hot to cool war materialize as suggested, the military command control problem will grow to resemble more and more closely the corresponding business, industrial, and governmental problems.

This paper discusses a problem domain in a way that is intended to integrate a number of problems too often seen only in isolation. In choosing among the levels of discourse at which it is possible to formulate such a problem, the criterion was to bring the course of the discussion tangent at some point to the body of interests of participants in system operation, who make the decisions; system analysts who must identify, classify, and assign the decisions; system designers who must implement the means for forming the decisions; and theorists and experimenters whose aim is scientific understanding. The terminology therefore may be somewhat mixed, but that is an acceptable price to pay for the attention of those workers in the diverse fields who have substantial contributions to make to the solution of the problems of human decisions in complex systems.

Discussion

JOHN LYMAN (*Biotechnology Laboratory, Department of Engineering, University of California at Los Angeles, Los Angeles, Calif.*): It appears that the con-

tent of this first group of papers has raised three classes of decision problems, as follows: (1) problems associated with selecting or "gating" information from the environment; (2) problems of collating information with that in human storage and establishing criteria for when the information is complete enough for a decision to be made; and (3) problems of organization and control of action elements to carry out the effective utilization of available information.

A further set of problems seems to be evident but was touched on primarily by implication rather than directly. This set includes the problems associated with selecting goals and setting values for the functions of the systems themselves. Adelson's paper was particularly relevant to this. It is becoming more and more manifest in our time that much that will influence human "ways of life" favorably depends on what humans decide to do with their environment to modify it in various dimensions. Decisions about such matters depend on processes about which little appears to be understood. Accordingly, this appears to present a particularly challenging area for research exploration.

Human observers are known to perceive situations on the basis of past experience and instructions for observing. Following this argument, it is speculated that radical effects on decision outcomes may be shown to be related to perception along a "value" or personal criteria continuum for both simple and complex tasks. It is suggested that any complete theory of human decision processes must take account of the parameters associated with the formation of human judgment criteria, as well as the so-called "objective" facts.

Part II. Theoretical Approaches to the Decision-Making Problem

LEVELS OF INTELLIGENCE IN DECISION MAKING

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Introduction

The advent of complex man-machine systems has focused scientific attention on the general properties of automata. Determination of realizability, channel capacity and other essential limiting factors have been accompanied by the development of rather sophisticated techniques for the analysis and synthesis of machine† systems. There has been, however, a distinct lag in comparable technology relating to the designed use of the human operator to work as part of a man-machine team.

This deficit becomes even more apparent when the operator's assigned task is more than simple tracking or other conditioned reflex decision behavior. Automation, the replacing of lower level human functioning by that of machines, is becoming limited by the lack of understanding of what the human does in arriving at a decision. Today the designer includes human operation in the system *only* when his knowledge of the problems that the system will encounter is incomplete. He makes up for this unknown, and the unexpected, by specifying "a portion of himself" as part of the design. The only true justification for including man in modern equipment systems lies in his unique value as a decision maker. The greater the degree of automation, the greater the burden of responsibility that rests upon the remaining human decision making.

It is considered worthwhile to attempt to quantify the decision-making capability in order to characterize human performance better and, as such, offer a scale useful in the design of future automated systems. Complex system design remains an art until a set of measurements becomes available that will describe equally well the performance of both men and machines. Indeed, such measurements would also prove of significant value in the design of purely mechanized systems or subsystems to meet given logical requirements most efficiently.

The external behavior of any automaton organism (man and/or machine) can be fully described in terms of its input stimulus and output response, given a particular history of previous experience and behavior.[1] Such transduction may be viewed as being a result of internal decision making. The purpose of this paper is to offer a quantitative measure for the intelligence of any decision-making transduction. Although the terms intelligence and decision making will be generalized far beyond their subjective connotation, it is

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† The term machine is taken to indicate any artificial mechanization, including electronic and other types of systems.

desirable that they agree with the more usual intuitive notions. The intended measurement must prove to be of far greater value than the intelligence quotient (IQ)* in describing men, machines, or man-machine combinations.

The intelligence of specific decision making is a property of the particular transduction and, as such, it is bounded, but not determined, by the physical nature of the automaton and the system in which it is enclosed. That is to say, an automaton-organism need not operate at its highest level of intelligence and, in fact, might reach this level only on rare occasions. The amount of intelligence that is brought to bear is dependent upon the nature of the system within which the automaton finds itself. If only the simplest decisions continually satisfy the environment it would be unwarranted (and possibly even unwise) to exercise greater intellect. In general, then, the level of intelligence is directly related to the decision making that is accomplished.

Before considering an absolute measure for intelligence, it appears necessary to determine an ordinal scale of the logical complexity of decision-making behavior.

Deductive Decisions

Deductive decisions can be carried out with complete certainty. Deduction is accomplished when a particular case is drawn from the general rule as a result of some inquiry that falls within the class covered by the rule. To give the classical example: Socrates is a man; all men are mortal; therefore, Socrates is mortal. In more modern terms: given an input, x_i, and an appropriate operator, F, the result y_i is determined by $y_i = F(x_i)$.

At the lowest level of logical complexity is the special deductive case of "no output-response" transduction. The automaton receives an input stimulus but continually remains "silent." It is important to note that the decision belongs only to this "mute" class as long as the lack of output is not a coded response in some manner. In general a deductive decision maker is preprogrammed to carry out its operation in a deterministic manner, translating each of the possible inputs into some particular related output. The automaton attempts to do only what it has been programmed to do. No matter how complex the actual transduction may be—whether it be multivariate, nonlinear, or the result of operation upon the memory—it is possible, at least theoretically, to determine what the output should be from knowledge of a blueprint of the machine. All variability in the output is due solely to noise arising at any point in the transduction.

To illustrate, we may cite the simple mechanical control system of a small aircraft that receives input signals from the human operator and transduces them into control-surface movements. Each control-stick input should always result in the same aileron- and rudder-position output. Any gross variation in this transduction is the result of noise and yields degraded performance.

The resulting change of flight path of the aircraft is, in a similar manner, the outcome of a purely deductive decision by the aircraft itself, the composite

* IQ is a relative psychometric measurement of the performance of a subject with respect to a standard test under "laboratory" conditions, the average IQ being arbitrarily taken to be 100.

of both the control surface input signal and the inertia memory of previous control actions and flight. The aircraft response would be exactly predictable in the absence of noise and here, again, the only result of imposed environmental or internally generated noise is uncertainty in the outcome that degrades performance.

It is important to realize that deductive decision makers need not be passive. In fact the human operator depends to a surprisingly great extent upon conditioned-reflex decisions that occur below the level of conscious mediation. Acquisition of such conditioning is certainly worthwhile in order to achieve rapid and dependable response to certain important stimuli (for example, emergency signals). Their utility, however, in no way affects their logical complexity. Such simple input-output decisions might best be called "moronic." Obviously any logically simple system of automata composed of members that make decisions only at the moronic level of logical complexity may also be classed as moronic. Regardless of physical size or structural complexity, the preprogrammed input-output transducer remains the maker of moronic decisions, as represented in FIGURE 1.

At a higher level of logical complexity there are deductive decisions that intend to optimize performance with respect to some given set of criteria. The

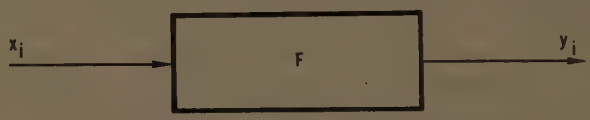


FIGURE 1. Moronic deductive decision.

automata accepts information from two separate sources—the reference signal and the sensed environment—and combines these under some given rule (set of criteria) to yield the particular "best" output response. The information derived from the environment is related to previous values of the output control signal and thus provides a closed loop through the memory of the environment.* The *optimizing* decision maker compares the two input signals in some manner, thus forming a model of the transduction that must yet be accomplished in order to maximize the value of the given measurement as defined by the set of criteria. The decision maker is structured to be capable of accomplishing only a certain set of control signal responses. It selects one of these as being most appropriate, based on the particular disparity between the intended environment and the actual environment as viewed through the criteria.

In general the optimizing decision maker is composed of a model and some implementing transducer, configured in either of two possible ways. As shown in FIGURE 2, the optimization is accomplished by designing the model of the desired transduction so that a properly revised reference signal is generated and furnished to the implementing moronic transducer. The usual closed-loop servo system falls within this class of optimizing decision maker. The error is formed by taking the difference between the reference variable and the

* The real world memory may be simply in the form of inertia or structural integrity.

status of the controlled variable. This error becomes the input to the moronic implementing transducer. Such automata continuously attempt to minimize the error under the tacit assumption that this will also maximize the value of total transduction.

In an alternate configuration, shown in FIGURE 3, the model is used to determine an internal signal that will modify the implementing moronic transduction so that its operation on the actual reference signal will yield directly the desired output response. This configuration may be called an adaptive optimizing decision maker. In recent years there has been considerable interest in the design and use of such adaptive servo systems.

Various combinations of these two types of optimizing systems can yield somewhat more complex automata that use the model not only to modify the

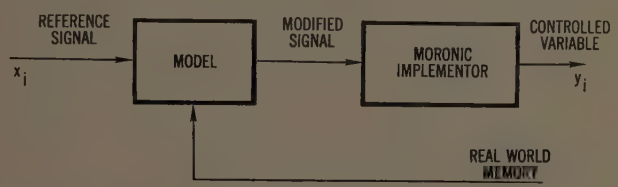


FIGURE 2. Optimizing deductive decision.

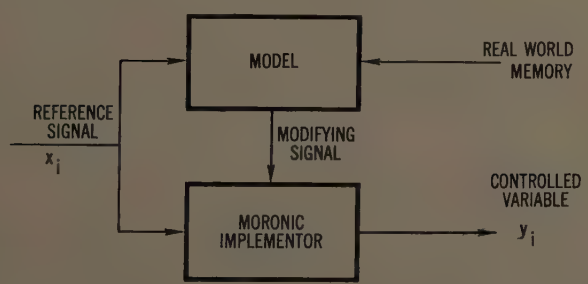


FIGURE 3. Adaptive optimizing deductive decision.

input signal but also to modify simultaneously the transduction of the implementing output device. Such design becomes particularly appropriate when limiting constraints are placed upon the signal range as well as the variability of implementing transduction behavior. It is not uncommon to come upon multilevel closed-loop systems designed so that a hierarchy of optimizing decisions enclose one another. Furthermore, complex mechanizations may encompass a number of importance-weighted parameters and maximize the resulting figure-of-merit in order to determine an optimal corrective action. Regardless of the complexity of the physical configuration there results a system operating at the same level of logical complexity: that of deductive optimization. In general, any optimizing decision may be viewed as composed of separate model-determining and moronic-implementing subdecisions.

Deviation of the output-solution from that determined by the program of a deductive decision-maker is the effect of noise. Usually this results in degraded performance; however, it is possible to offer specific instances in which

the introduction of noise may aid decision making. To illustrate, consider an optimizing decision maker operating within the noiseless domain of pure numbers. The imposed reference criterion is that the output solution be a rational number lying between 6.5 and 6.6. The automaton is programmed to use the four simple arithmetic operations on any of the whole numbers ranging from zero to nine as "sensed" from the environment. Successive combinations are accomplished through a programmed orderly procedure. In each case the related error is determined from the difference between the created number and the desired output.

The automaton initially examines each of the whole numbers and finds that a satisfactory solution is not yet available. It therefore proceeds to generate combinations forming a new set of possible solutions, these being $1 + 2$, $2 + 3$, $3 + 4$, and upward. These also fail the test; therefore the other arithmetic operators are brought to bear in order, first on digrams, then on trigrams, and in ascending succession. The set of possible solutions generated is infinite in extent but includes only the denumerable set of rational numbers.* Eventually, a valid solution will be realized, regardless of the specific procedural order of programmed investigation.

It is easy to find a simple criterion that would prevent a valid solution from ever being realized, even within this large but specialized domain of generated alternative solutions. For example, suppose that the requirement had been that the solution be an *irrational* number lying between 6.5 and 6.6. It is obvious that no such number will exist within the generated set of possible solutions. At this point it is worthwhile to consider the addition of analog noise in the construction of the set of possible alternative solutions. Each time a number is taken for combination it is slightly different in value and, in fact, some variability is also characteristic of the combining arithmetic operators. As a result, each generated combination may be slightly different from the noiseless result and from any previous attempts to accomplish the same combination. Notice that the addition of noise has released the denumerable set of possibilities into a domain of nondenumerably infinite extent. The same deductive optimizing review of the possible alternatives can now be accomplished with success, since the entire set of irrational numbers will eventually be generated.

It is interesting to note that for any such decision maker there is some optimal signal-to-noise ratio in order to attain a valid solution within a minimum time of operation. If the noise level is too great with respect to the signal, a great diversity of alternatives will be rapidly generated, covering a wide domain far afield from the original given alternatives. Such "radical" adduction may furnish a large number of solutions that require the expenditure of considerable time and effort in examination prior to finding a successful solution. In fact it may be that such introduced diversity may skip over the desired solution, thereby greatly increasing the total cost of its final determination. On the other hand, too low a noise level will only modify the sensed data in a most conservative manner, thus providing a slowly increasing domain of possible solutions that is more carefully surveyed as the deductive process continued

* The solution might first appear in the form $(8 \times 9/6 + 5)$.

in its review. Obviously there is some "best" level of noise that will generate the required solution at a minimal cost of time and effort. Noise is multidimensional. Each of the parameters within which a decision maker operates may be noisy so that, regardless of the provided constraints, it always remains possible to find a solution to a given problem through the useful use of noise, provided the given problem is itself consistent and possible of solution in the real world.

At the lower levels of structural complexity noise is almost always a degrading factor. It has been shown, however, that at a higher level of logical complexity noise can serve a useful purpose by releasing the domain of those alternatives that are available for consideration. The automata may be said to have "created" or "invented" new solutions that may then be recognized as valuable by the ensuing deductive process. By analogy it might follow that the process of human invention is the result of noise internal to brain functioning coupled with careful deductive examination to determine which of the generated products are of immediate utility (that is, which generated products satisfy the constraints of "necessity").

The real world is always noisy. The likelihood that this noise will significantly increase the utility of a mute decision is nil. The probability that the result of a noisy moronic decision will happen to fall within the desired range of parametric values is somewhat greater. The search for a valid decision by an optimizing automaton allows even greater opportunity for the useful use of noise, since the feedback provides a deterministic influence toward reaching the desired solution. It appears reasonable to suggest that the greater the logical complexity, the greater the possibility for the use of noise to facilitate reaching a successful decision.

Abductive Decisions

Abductive decisions usually have some essential uncertainty and yet this type of decision is common to everyday experience. Abduction is accomplished when a particular "causal" factor is to be determined as a result of some given outcome "effect" and a governing rule. In the classical idiom: Socrates is mortal; all men are mortal; therefore Socrates is a man. Perhaps, but Socrates might also be a dog. That is to say, given the resultant, y_i, and the operator, F, then the causal factor, x_i, is presumed to be determined by $x_i = F^{-1}(y_i)$. Even though uncertainty prevails the decision maker may have no better course of action than to use the given rule directly in an inverse manner. Possibly the greatest danger lies in the decision maker's not "realizing" the degree of uncertainty that is inherent in his abductive decision.

Evidence is presented. The jury is presumed to be sufficiently mature and experienced to know patterns of human behavior. Based upon these "rules" and the provided evidence, these "good men and true" must abduce the most probable causal factor and declare innocence or guilt accordingly. Circumstantial evidence has certainly been known to lead to false conclusions.

The radar operator sees a particular pip on his scope. His task is to determine if this pip represents a real target or is just the result of noise. The rule is provided in terms of his experience using similar equipment systems under

similar circumstances. The operator makes an abductive decision, and his decision remains dependable only to the extent that his "rule" is useful to discriminate signal from noise and also covers the actual real world situation encountered. His decision task may be viewed as one of pattern recognition. Neglect of the noise level can easily demolish the value of decision making.

Taking a measurement is an abductive decision. A thermometer provides direct evidence only in terms of the amount of expansion of, for example, mercury; the implicit rule is the particular structure of the tube and bulb. The decision maker uses the indicated scale value and the "given" rule defined by the explicit scale on the tube in order to abduce the corresponding temperature. Even the simple measurement of length is indirect, as it is in terms of the number of arbitrary units that cover the distance. An abductive process is needed to reach the desired conclusion. It is easy to find a wide diversity of measurements that, by being indirect, require an abductive decision because pattern recognition is only one form of measurement.

Inanimate mechanisms may be programmed to accomplish abductive decisions, as demonstrated by a number of recent experiments wherein a digital computer constructed logical proofs for stated theorems given the rules of manipulation and previously proved theorems.[2] The adjoint method for investigating a control system is also an appropriate example of abductive logic.[3] The adjoint of a time-varying system is that system whose impulse response, considered as a function of measurement, is equal to the impulse response of the original system when taken as a function of the time at which the impulse is introduced. A well-defined procedure exists that allows description of the adjoint of a system. The result can then be explored, through the use of an impulse function, to reveal its transfer characteristics and the probable causal factors for any known response.

Such abduction may be viewed as an array of deductive subdecisions, as shown in FIGURE 4. The general rule is translated into its inverse, assuming this to be possible; the result is used to program the moronic deduction that transforms the data at hand into some possible causal factor. This latter subdecision might be viewed as a constructed model of the real world process under investigation. There is an interesting similarity between the logical configuration shown in FIGURES 2 and 4, each appearing as a reflection of the other.

It is common to require optimal abduction, that is, the determination of the most probable cause for a given effect. This can be provided by following the above-described abduction with an optimizing deduction to select one that corresponds to the greatest value from among the set of generated possible causal factors. The diversity of this set might relate to an unknown constant of integration, in the event that the output of the real world process is described in terms of a convolution integral using the given kernel.

Abduction need not relate to time-ordered factors linked by causality. In the sequential case, however, it is usually found that the greater the span between the reference and observed time, the greater the abductive uncertainty. Stated in another way, the point of minimum entropy in the perceived world corresponds to the present.[4] Entropy increases as the "view" extends farther into the past or the future.

Another subdecision in the abductive process may relate to the required minimum level of uncertainty of the solution.[5] If there exists such a constraint, it may prove appropriate for the automata to review the available data until a sufficient amount accrues to satisfy the requirement for valid abduction. Such an uncertainty estimate is most likely to be deductive based upon a rule derived from past experience.

A very large number of abductive decisions fall in the class of statistical decision problems. The automata-organism is presented evidence on some available sample and upon this basis must determine the nature of the most probable parent population in terms of some choice of hypothesis.* The literature on this subject is extensive.[6,7] The field of signal detection theory has developed upon this foundation. A rather extensive normative theory has been favorably related to empirical evidence regarding human detection of signal in noise.[8,9,10] Game theory and dynamic programming cover an abductive domain wherein the decision proposes a best course of action (causal factor) under a given set of cost criteria and a desired ultimate condition.[11,12] Detailed discussion of these topics is considered beyond the scope of this paper.

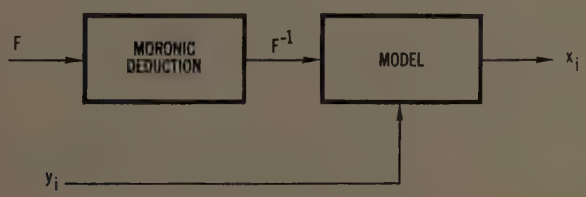


FIGURE 4. Abductive decision.

The use of heuristics in abductive decision making can prove to be of distinct value. The "reasonable" choice of program can produce a set of possible solutions that will more quickly satisfy the imposed constraints. The heuristic decision maker may benefit from past failure as well as from past success. The decision may have proved to be negative, and yet it might be possible to extract some elements of positive value in order to improve future decision making. Heuristics relate to the program of decision and, as such, may be considered as an overview "rule" derived from the memory of past experience.[13] The application of the heuristic rule compounds the given rule and usually proves to be of benefit (although it is quite easy to construct mathematical problems where intuition fails). Decisions that determine the nature of the rule under which abductive decision is reached are known as inductive decisions.

Inductive Decisions

Inductive decisions always remain uncertain to some degree. Induction is accomplished when evidence of at least two specific data points is available and there is a desire to generalize upon this basis to some governing relationship. It is impossible to find a method of constructing exact generalizations

* As long as the decision considers which of the available hypotheses are true the decision is abductive. On the other hand, the generation of hypotheses is an inductive decision.

which will always remain valid. Furthermore, the apparent validity of one particular logical pattern in no way offers proof that there may not be some other equally valid and perhaps even more appropriate pattern. To quote the classical idiom again: Socrates is a man; Socrates is mortal; therefore all men are mortal. Reference to Venn diagrams makes the uncertainty condition immediately apparent. In more modern terms: given x_i and y_i, find the F that might have caused this occurrence or relates the available evidence.

Probably the fundamental problem of inductive decision making relates to the establishment of categories. The two given pieces of evidence are alike, but in what manner? The young child views an array of coins. He separates those of greater "value" from the rest by choosing the shiny ones.[14] His decision indicates an implicit generalization "Shiny coins are 'better' than dull ones." Without any given categories the inductive organism must generate its own set of classes by whatever discrimination means are available. MacKay[15] has suggested how classes may be generated at increasing levels of abstraction through deductive logic operating upon the ability to discriminate some fundamental property. The ability to erect a classification at even the lowest level may require that the automaton possess a model and choose to accomplish a comparison subdecision at least at the nominal level.

Let it be presumed that an elementary automaton-organism can distinguish only the existence of light (versus dark) in its environment. Memory of the immediately past state provides access to "has the environment changed?" Correlation of such a perceived light-intensity change with exerted effort might constitute the basis for the discrimination of movement (two classes: moving versus not moving). Movement is a vector and may be classed with respect to direction and, through the use of memory, with respect to patterns of direction. Patterns of patterns lead to higher and higher levels of abstraction.

In view of the stated requirement for at least a minimum of two data points it is interesting to consider the original induction of a newly formed automaton-organism that is as yet memoryless. Let it be assumed that no outside instruction or program of classes has been provided. Is it possible for such a system to accomplish inductive decision? An affirmative view can be offered. All signals that can be distinguished from noise have some internal redundancy. Only through such redundancy is detection possible (abduction). Accordingly even the simplest signal can be divided into at least two data points, thus permitting some generalization (possibly regarding its source).

The elementary subdecisions for induction may be viewed as shown in FIGURE 5. Two class-model deductive transducers receive the available information from either the sensed real world, the memory, or both. Their outputs represent only the class characteristic of the input signals. These are then compared to determine equality within the range of discrimination of the class models. The result of the inductive decision is an hypothesis with respect to the entire population from which these signals x_i and y_i were originally drawn. At higher levels of abstraction the class-model transducers become multidimensional, and comparison can be accomplished in a number of respects at the same observation. The result can reach any desired complexity of "hy-

pothesis" or "governing rule." Of course the greater the specificity in each dimension and the greater the number of dimensions viewed, the more likely it is that the inductive solution will truly represent the real world.

A more general set of conditions that make possible the existence of a classification system have been stated by Uttley:[16]

"1. Each input channel must be always in one of two states—activity and inactivity.

"2. The inputs must be combined in as many ways as possible—ideally in all possible ways.

"3. There must be a unit for every combination of inputs, which indicates that every input of the combination is active. A combination of a set of inputs is said to define a pattern of activity. The connections between inputs and units are called counting connections.

"4. Temporal patterns are to be distinct. Each input must pass through a period of delay; the output of each delay must provide a separate input to the system of indicating units."

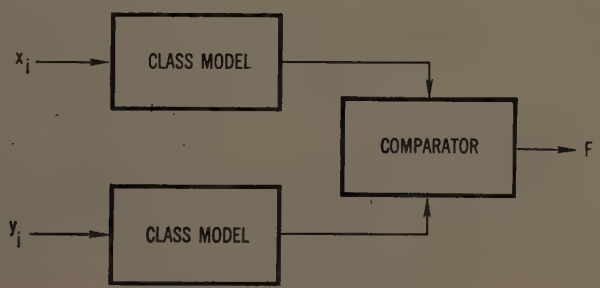


FIGURE 5. Inductive decision.

At present there is significant interest in heuristic programming. The term "heuristic" is taken to mean the accomplishment of "reasonable" decision-making behavior. At the lowest level it is possible to program adaptive behavior in such a way that the mechanism will continue to solve the given problem by a successively more and more efficient technique (the transduction attempts to attain greater value with respect to the given set of over-all criteria). Such a device is usually not dignified by the adjective "heuristic," this being reserved for automaton organisms that appear to accomplish the same end, but lack the explicit framework of an over-all criteria that justifies the modification of program. Note that whether the reprogramming is the direct outcome of the original structure, it corresponds to the construction of a new "rule" that may be said to correspond to a model of that transduction required "to satisfy the real world environment."

In living organisms decisions are rarely, if ever, reached on the basis of a singular parameter. In the inductive case it might be observed that a multiplicity of decision levels are apparent for even the simplest case. Everything of concern in developing a program, a rule, or a generalization is related to an entire hierarchy of subgoals of various utility weightings. It is suggested

that the heuristic human capability is a result of the concurrent generalization of the given evidence at various levels resulting in a set of "rules." If the decision reached on the basis of the lowest-level rule proves to be improper, there is no need to throw away the generalizations at all higher levels. These can be retained to aid in formulation of a new decision based upon the same or additional evidence (including the result that the previous solution proved to be of little or no value).

To illustrate with an oversimplified example: given the sequence of numbers 1, 2, 3, find the governing rule needed to generate the next digit. The decision maker generates the obvious 4; that is, the rule is taken to be the set of whole numbers, 1, 2, 3, 4 · · · . The examiner (representing the real world) provides a feedback that indicates this is not correct. The decision maker approaches the same data once again. No consideration is given to the possibility of changing the higher-level tacit assumption that the "rule" must be some number sequence, with letters and all other symbols excluded. The decision maker proposes 5 (the result of the prime number sequence) or 6 (the result of a Fibonacci set wherein each digit is the sum of the preceding two digits). Only when generalizations at this level of probability have been exhausted (within the domain of whole numbers) will the reasonable decision maker question the higher-level tacit assumption. Could it be that the given sequence of numbers was taken from an automobile license plate? Possibly the generalization should include letters of the English alphabet.

A heuristic program, then, requires the automaton to hold some *set* of generalizations as a solution to each inductive decision, even though its direct solution may be the result of only one member of this set. The set of generalizations covers various levels of abstraction; those at the higher level are usually carried forward to guide the reasonableness of the succeeding (and more probably successful) approach to the problem area.

These concurrently generated rules may each be taken as a model of that transduction required by the real world environment at its own level. As such they represent a set of normative self-images by the automaton. In life the rule that is actually drawn upon may not always correspond to the lowest level of abstraction. The organism might more naturally choose to utilize that level of abstraction at which the normative model and the real world are already almost in agreement, thus requiring a minimum of change. The similarity of such behavior to that of subjects under psychoanalytic therapy is obvious. The initial change of behavior serves to maintain the present ego status (the normative self-model).

There seems to be an underlying belief in scientific circles to the effect that the real world must be essentially simple. If a number of hypotheses fit the data then it is the simplest of these that is considered most worthy of attention. There are both good and bad aspects to this viewpoint. However that discussion is considered beyond the scope of this paper, but may be cited in passing here as an example of the basis upon which the particular generalization that is implemented can be chosen. Undoubtedly there are many other equally "rational" bases (note that this choice represents a governing decision that forms a preprogram for the generated set of rules "that allow heuristic reasoning").

To achieve a heuristic program, then, an automaton must have the capability of developing a number of models of its own behavior. This paper does not presume to summarize all aspects of heuristic decision making. Reference is made to the growing literature.[13,15,17,23]

Generalizations at various levels can be observed in the learning behavior of animals. As described by Hebb,[24] it is possible to train both a rat and a chimpanzee to look for food behind a white triangle symbol (FIGURE 6*A*). This learning may take as many as 200, 300, or 400 trials for the monkey, while the rat may demonstrate the learned behavior after only 10 to 20 trials (or for somewhat more difficult discriminations, after 50 to 80 trials). Testing of these same animals with other stimuli reveals that their individual perception of the white triangle stimulus was indeed quite different. The rat offers only random response to FIGURE 6*B*, *C*, and *D*, whereas either FIGURE 6*B* or *C* will elicit a positive response from the chimpanzee, demonstrating generalization at a higher level of abstraction. It is interesting to note that the two-year-old

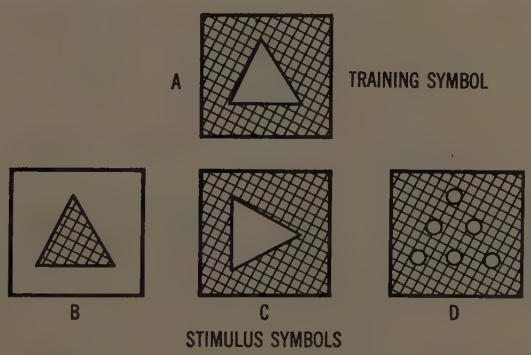


FIGURE 6. Training symbol and stimulus symbols.

human child will also recognize the training symbol in FIGURE 6*B*, *C*, and *D*. Possibly the set of generalizations may best be thought of as consisting of included subsets of the generalization at the highest level of abstraction. The consistency of these subsets may determine the stability of evidenced behavior.

Inductive decisions are most natural to all living organisms. The essential instinct for self-preservation leads to the desire for stability of the environment, that is, cognition and recognition of all observed entities. Cognition requires classification and, as such, generalization. The result is induction.

Projective Decisions

The scientific method, as indicated in FIGURE 7, may be viewed as a structure of the three syllogistic decision types. Those aspects of the real world that form the subject of investigation are viewed with an inductive decision process as resulting in some semantic description in terms of a hypothesis or a model. This model is manipulated into some more tractable form called a solution, through the use of a deductive syntactic transformation. Finally, the particular point in question is brought into focus: estimate the condition of the subject system at a particular time or as related to the value of some

other parameter. This pragmatic link is traversed through an abductive decision using the solution as the given "rule." The validity of prediction, postdiction, interdiction, interpolation, or extrapolation by the scientific method is dependent upon the error introduced in the triplet of decisions.

Autonomous Decisions

The autonomous decision reaches a higher level of logical complexity when an uninstructed automaton is released in some environment and finds it possible to determine its own problem. All organized systems have a singular short-term property in common: the tendency to preserve their state of organization. Living organisms clearly exhibit this self-preservation instinct; that is, they program their decision-making behavior to minimize the effect of a sensed threat to the environment and thus tend to maximize their probability of survival. Self-preservation is common to all organizations of living creatures. For example, governments provide standing military forces, com-

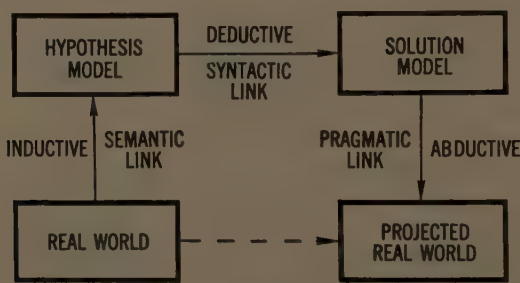


FIGURE 7. The scientific method.

munities have Chambers of Commerce, and social clubs take action for the maintenance of their existence as any threat appears. (The citizen's secondary instinct of procreative reproduction may be viewed as the primary self-preservation instinct of the society of which he is a member.)

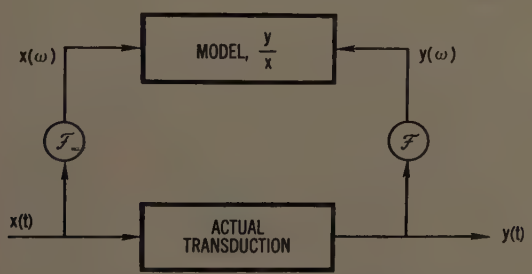
It is interesting to note that inanimate systems also exhibit a passive tendency to preserve their structural organization. Any mechanism is an organized embodiment: its very organization provides a cohesiveness that resists destruction; that is, energy must be expended in order to dismantle or destroy even a nonliving system. Taking the broadest possible viewpoint, this passive resistance to disruption forms a kind of "self-preservation instinct." Such "structural static inertia" results from the strength of such factors as bonding of the component parts, the information flow within the system, and the amount of memory.

Autonomous decision making requires the active pursuit of self-preservation. To accomplish this end the automaton must hold, let us say, some degree of awareness of its own operation, or some self-recognition. This can be accomplished if the automaton can generate a model of itself. For example, a model of only the spectral properties of the basic transduction is indicated in FIGURE 8. A transformation can be applied to both the reference and controlled

variables to yield their spectra. These are compared to form a model of the automaton consisting of a rational fraction polynomial that expresses the amplitude versus frequency behavior of the transduction. Under different circumstances it might be more appropriate to generate a statistical model of the transduction through the use of tabulating translators permitting the generation of a stochastic representation.

Regardless of the technique by which this model is generated, it is never possible for the model to be an exact replica of the transduction process. There is always some delay in translation and, in the statistical model, the essential assumption of even short-term stationary status prevents the correspondence from ever being complete. Any noise introduced immediately corrupts the model, thus obviating an exact representation.

Humans behave in a somewhat similar manner in their use of models of themselves. Human mental models are certainly not an accurate representation; in fact, there is not even an attempt to provide full representation. The



$\mathcal{F}$ -PROVIDES SPECTRUM OF SENSED VARIABLE

FIGURE 8. Generation of model of transduction.

human wants a simple "picture" of his own operation in order to permit simple reasoning in the face of a particular aspect of his environment. It would admittedly be inefficient to approach each task at hand subjectively with a complete representation of all aspects of one's self, even if this could be attained.

Let it be presumed that some portion of an automaton is devoted to the generation of a model of its over-all transduction. As time proceeds this model will reflect the degrading effects of noise, both internally generated and externally incident disturbance. With this model available as evidence of this "real world" situation it is possible to superimpose an optimizing deductive decision process wherein either the initially programmed (genetically given) ideal model of the transducer or some recall of a previous model state is used as the reference signal. The result of this comparison of the reference signal to the model of the actual transduction in itself forms a new model of the difference in level of self-preservation. This higher level model provides a separate supervisory function that monitors the degree of system success with respect to survival.

FIGURE 9 indicates such an automaton. Many other possibilities exist,

including the comparison of the actual transduction (as represented by the model) to a predicted model that forms the reference signal. The prediction could be carried out through a rather complex decision procedure that is severely sensitive to the relative conservatism or radical attitude in terms of the relevant noise.

It may be speculated that the essential difference between man and the lower forms of animal life lies in his capability for apperception. A multiple structure of models allows him to think about what he is thinking about, and it is felt that herein lies the key to abstraction. It seems reasonable to presume that an inanimate automaton could be designed to include models at various levels and in such a way accomplish an analogy to thinking about thinking. The cortex of the brain is well known to form an analogical model of the human sensory-motor transduction. Furthermore, additional related models have recently been identified within other portions of the brain. Could it be that the identification of such a sequence of models might yield insight into the inherent human thought capability?

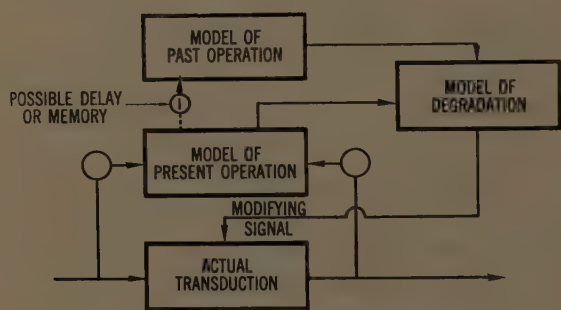


FIGURE 9. Autonomous decision.

The key to achieving autonomous decision making may prove to be the separate use of a portion of the decision-making system to form a comparative model of its own change of status. It is on this basis that the program of the basic transduction may be modified to prove more efficient with respect to the encountered environment or a change of criteria. It is the noise acting on the automaton that furnishes the motivation for higher levels of performance.

The Decision "Continuum"

In the human operator it is indeed difficult to determine discrete levels of complexity in decision making. An inexperienced driver may accomplish steering control at the moronic level and white-line tracking by deductive optimization. As experience accrues the tracking task becomes more and more one of conditioned reflex until it too finally reaches the less complex status of a moronic transduction. It is generally true that the learning process succeeds in replacing higher levels of complexity with those of lower level to accomplish the same required result. More complex decisions of higher logical complexity are called upon less frequently. However, the result of

their action may be expected to be more dramatic, and their occurrence is therefore more obvious. There appears to be no reason to believe that an upper limit of logical complexity has yet been reached by the human operator in his decision making.

In 1936 Turing[25] succeeded in specifying a "universal machine" that possessed but few internal states and external transduction abilities. However, it was shown that such a Turing machine could be programmed to reproduce exactly the behavior of *any* physically realizable machine as long as no limitations were placed on the required time or memory capacity. Human operation is of some finite, though large, number of internal states. It therefore follows that in theory, at least, it should be possible to duplicate exactly human decision making provided, of course, sufficiently detailed information was made available describing the human transduction. Such a programmed Turing machine might even exceed the human capability with respect to equivalent logical complexity. Computers have already performed certain heuristic problem solving for which no algorithms are available and reprogram themselves (learn) only on the basis of experience.

The Intelligence of Decisions

In 1873 Jevons[18] stated "Economy of mental power may be considered one of the main conditions on which our elevated intellectual position depends." Intelligence may be defined as the absolute probability that a proper transduction will occur for any particular given task. The more efficient the automaton is in time and effort, the greater will be this probability (reliability). It follows that intelligence of decision transduction must be between zero and unity for any task (defined in parameters and time tolerance). Perfect intelligence is unrealizable due to the uncertainty caused by real world noise. Zero intelligence is ruled out by the pure chance possibility of any desired occurrence. Even completely random unstructured automata might happen to create the required output that corresponds to an acceptable solution. This probability, granted, is extremely small, but it always remains greater than zero. Specific calculation techniques are available from work done in statistical mechanics[19,20] and stochastic processes.[21]

The above-described ordinal scale of logical complexity of automata may be viewed as a set of alternate hypotheses available to solve the problem at hand. It may be possible to choose too simple or too complex a program. In fact, certain problems may not be solvable below some level of logical complexity. Only the optimal level of complexity provides the most intelligent decision making.

Human intellect appears limited by the efficiency of self-programming and reprogramming. Today this property can be measured only in terms of the intelligence of accomplished decisions. The near future will bring increased unity of the human operator and his computer tool. As a result the human may achieve greater reliability of decision than was previously possible. Is it not possible that the self-reprogramming of a computer could prove to be even more efficient in choice of program and reliability of execution—and thus far surpass the intelligence of its human counterpart?

Practical determination of the intelligence of any particular decision may prove to be a difficult task. The variability imposed by such factors as noise and transduction change by learning or fatigue all combine to prevent exact reproducibility. Only through recourse to statistical techniques is it possible to estimate the desired probability of correct solution. The larger the sample of available experience within a particular class of decisions, the more accurate will be the estimation of individual decision intelligence. The simple correspondence of intelligence to decision reliability makes it possible to utilize well-known analysis techniques. Complex systems may be reduced to functional schematics and the related reliability equations. In the case of man-machine systems, machine reliability may best be interpreted in terms of an "intelligence level." This correspondence may offer new insight into optimal synthesis of future man-machine systems.

Conclusion

Man is increasingly dependent on the technology he has created. Devices and techniques that originate as luxuries rapidly change into necessities. Expanded suburbia, for instance, makes the automobile an essential part of life. Job assignments become more and more specialized by using the "tools" that extend human capability. These reach out in every direction, including amplified force, greater communication range, and faster transportation. Not the least of these relates to data processing, allowing the replacement and reallocation of the human mental capability. Progress toward automation is no longer bounded by the limitation of the available hardware; rather, the design of efficient complex man-machine systems rests upon the proper determination of assigment for each of the decision-making elements.

To permit this advance it is considered essential to provide a set of adequate descriptive measurements for the decision tasks to be accomplished throughout the system. It has been the purpose of this paper to offer but one such measurement with respect to the often used (and misused) term "intelligence." It is not intended to imply that the proposed quantitative scale offers the sole valid approach. There may indeed be many useful techniques for measurement that deserves extensive exploration. If this paper serves to stimulate inquiry in that direction it will have performed its mission.

The near future will see an even closer relation between man and machine in the form of a "man-computer symbiosis."[22] The human operator will depend upon the computer to a large extent as an increase in mental capability. The machine will be referred to for technical manipulation, memory and information retrieval, language translation, and many forms of complex problem solving. Such man-computer teams might exhibit decision-making behavior at higher levels of intelligence than can be attained by man alone. Is it possible even to explore this realm of intelligent behavior from the limited vantage point of the present?

Considerata

The above discussion leads most naturally to certain fundamental questions that deserve specific consideration. It would appear that a noiseless decision maker can certainly contribute no new useful information, even though it

might yield decision outputs of much greater complexity than the given data at the time for decision. Thus additional information may be a product of the previously collected data, as stored in a memory or the essential structure of the device. That is, it may even be possible for the decision-making automaton to convert some of its own complexity into the output solution at no cost to the structure if a nondestructive readout is available. The term "new" information then must be considered to include only information that has *never* been available to the decision maker, including its own generation-design.

Let us now consider the effect of noise introduced during the decision-making process. As indicated in the above section, the extraction of useful information can be achieved through the action of a suitable filter devised to interpret properly the given set of criteria constraints required for successful decision. It is possible to challenge this formulation by saying that there is really no "new" information, since the filter is truly adding the information by selectively operating on the random noise. It is the selection of useful versus nonuseful pieces of random data that truly contributes the added information. Upon this basis the structure (of the filter) is contributing the additional useful information. In a sense, this point is analogous to the problem of Maxwell's Demon, which contributes the energy of selection in the famous information theory problem.

Is it possible to find a selection filter that could operate without having been programmed for the criterion of success? Stating the question in another and more general way: Is it possible to find a device that will classify without being programmed to do so? Since any inhomogeneity within the device may result in a separation of random data, it is necessary to start with the concept of a completely random structure. Only such a "device" can be said to be truly unprogrammed. The molecules within the system are excited by the ambient temperature, but in no way do they indicate any individuality.

At any particular point in time the fluctuations within the structure prevent its perfect inhomogeneity and, as a result, the device may selectively filter the inherent noise and effect the output solution. Such filtering would not prove useful since the particular nature of the filter would change from moment to moment in a random manner. The problem must, therefore, be restated: Is it possible to find an initially random structured device that will attain and *retain* the ability to filter selectively? Is there the possibility of a randomly organized structure becoming classified (structured) within itself and retaining that particular structure? If the answer is in the affirmative, then induction (which results from comparative classification) may truly be achieved by an inanimate machine. If the answer is in the negative, induction can be accomplished only on the basis of some requisite programming of a machine that instructs the set of classes of concern or some primitive set of classes from which the machine can deductively derive the desired set for the immediate problem at hand.

In order to answer this question it is necessary, first, to define more carefully the term "retain." For how long must the structure be other than random in order to prove a useful selective filter? Certainly the happenstance of classes must last for at least the time required by the decision process. If,

however, there is no a priori knowledge of when the decision will be called for, it would be necessary for the classifying structure to tend toward some stability. That is to say, this requirement would be satisfied only by a device that tended toward a particular ultimate classed state, but this appears to be in direct conflict with the second law of thermodynamics, which indicates that all systems will tend toward maximum entropy, toward greatest homogeneity.

This "law" of thermodynamics must be viewed with caution. It states that the system's increase in entropy is a statistically defined trend, but there is no requirement, for example, for monotonicity or for internal irreversibility of energy exchange. One of the defining characteristics of a living system is that it decreases its entropy; that is, it increases its structural complexity at the cost of a less-structured environment. Furthermore, even a nonliving system can have subsystems that, for some limited period of time, may operate in a completely reversible manner.

Returning to the basic question, it would seem that generally no mechanistic induction is possible (unless the entire system were living). The statistical dictates of the second law of thermodynamics in no way prohibit life, but they do place statistically flexible, finite bounds on the time period that may be used. In the specific instance of the automaton system it would appear possible to achieve classification without special programming for sufficient time periods to allow useful results. A living organism, for example, may fall into this class. This particular state measures zero over the infinite time domain.

There still remains a fundamental point. Only a "useful" selection filter is of any value. The probability that the random retained structure will prove to be "just right" is of negligible value. Only some communication from outside the automaton can offer the required guidance to avoid undirected induction (the extraction of useful data from noise is itself truly an inductive process, even though this was subsumed under a gross category of deductive decisions in the section above for a tutorial purpose). External communication can take place only through the identification of a class system (since this is a prerequisite for any communication system). Such instruction might just as well dictate the structure of the filter and not depend upon the availability of any self-structured state. In this case the induction is once again truly that of the external "instructor" and not of the filter, and the argument has returned to its starting point.

To summarize at this point: the logical process of an automaton can be discussed only if the system that *is* the automaton is clearly delineated. The philosophical inquiry touched upon should not impose any impediment to the use of the indicated classes for the description of decision making. It is felt that such identification serves a useful purpose in directing the mathematical inquiry regarding such factors as effectiveness, efficiency, and intelligence to facilitate optimal use for the available decision-making automata.

The purpose of this discussion has been fulfilled if the fundamental questions are made apparent. Too often such basic inquiry is avoided on fear of treading on the "metaphysical" or of opening controversy. It is only when ignorance is discussed that any *new* contribution can result. Avoidance of uncertainty prohibits progress.

Acknowledgments

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PHYSIOLOGICAL IMPLICATIONS OF PSYCHOPHYSICAL DATA*

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Introduction

In this paper I am presenting an approach to the study of psychophysics that includes the use of mathematical models. While the individual experiments can each be interpreted as a contribution to an area in psychophysics, the program can be interpreted as an experiment testing the feasibility of a profitable use of a particular scientific philosophy in the study of an area of psychology.

This philosophy involves the use of mathematical models in a manner similar to that of the physicist. It employs, first, the use of simple models describing the relations of only a few observable variables. The measures describing the relations among these variables may be measures of variables that exist only as force and gravity in physics and that can be measured only by observing their consequences. As measurement techniques become more nearly precise or, as the conditions of the experiment are extended to include a wider range, it will likely become apparent that the model had been adequate only because it was tested with coarse measurement over a restricted range of conditions. At this point it will be necessary to expand or modify the model to include the new data.

The expanded or modified model should still include the original model as applied to the limited conditions. FIGURE 1 is intended to illustrate this point. The large space is one that includes all possible data that might be relevant to a particular area of science. Many of the points in this space will never appear as data for they are contradictory to the "truth," whatever that may be. The shaded area in the center represents the data at hand. A model describing these data could be expanded in any of a number of directions. The large areas *A*, *B*, and *C* represent models, each of which includes the original model. If the next datum collected turns out to be at point 1, model *C* can be eliminated from consideration. If it is collected at point 2, *A* and *C* can be eliminated. Either point, however, requires an expansion of the original model.

We have tried to work with a general model that includes all of the points in the space. It is the general model of testing statistical hypotheses. The model can be expanded to specify the data that must appear if a sensory threshold exists, if Luce's[3] axiom 1 is valid, or if the observer is intelligent. The general theory can be made specific in a number of ways; it is an experimental question that determines precisely how it should be made specific.

I shall trace one phase of the history of this program, showing how we introduced the concept of memory to modify the model to incorporate a wider

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range of conditions. At times I shall try to point out some of the physiological implications. However, one must not forget that I am talking about concepts that are unobservable, and the physiologist may find himself faced with similar problems to those of the physicist chasing electrons: he may have to devise physiological cloud chambers so that his electrons will create an observable phenomenon. The nerve impulse may not be sufficient.

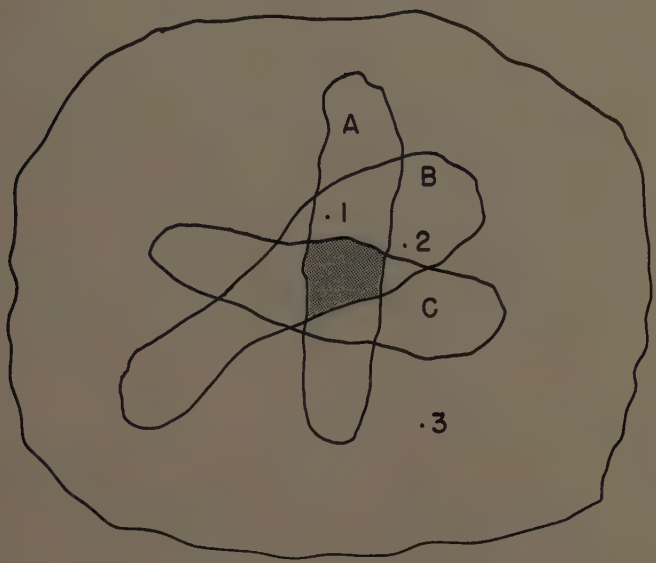

FIGURE 1. Theory space.

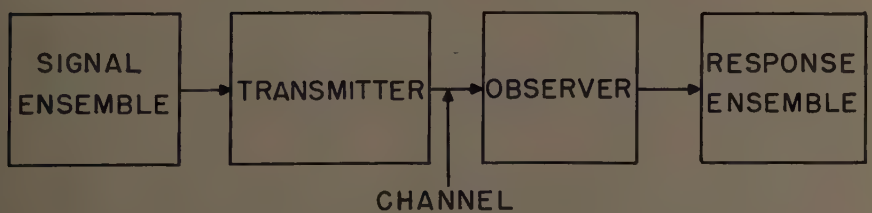

FIGURE 2. Block diagram of a typical psychophysical experiment.

What is a Psychophysical Experiment?

Psychophysics is the study of the relations of psychological quantities to physical quantities. A block diagram of a typical experiment is shown in FIGURE 2. The experimenter designates an observation interval in time, during which one of the signals from the ensemble is transmitted over the channel. At the end of the observation interval, the observer is asked to choose one of a set of responses. The joint event of the signal transmitted and the signal received (as indicated by the response) is recorded.

What are the Measures?

In the simplest type of psychophysical experiment, in which a particular signal or nothing is transmitted during the observation interval, the data may be summarized as illustrated in FIGURE 3. In this figure, the rows represent the transmitted events as modified by the channel. If the signal is transmitted, the observer's input is the signal as randomly perturbed by the channel. Here we are calling this input "signal plus noise." If nothing is transmitted, the observer's input is labeled "noise."

The columns are the observer's responses. "Yes" means that he considers the input to have arisen from signal plus noise, while "No" means that he considers the input to have arisen from noise alone. The data can be summarized as percentages that, in analysis, are assumed to be estimates of probabilities. Two conditional probabilities are estimated: the probability that if the signal is transmitted the observer's input will lead to a "yes" response,

	YES	NO
SIGNAL PLUS NOISE	n_1	n_2
NOISE	n_3	n_4

$$P_{SN}(A) \approx \frac{n_1}{n_1 + n_2}$$

$$P_N(A) \approx \frac{n_3}{n_3 + n_4}$$

FIGURE 3. Data summary of a typical psychophysical experiment.

$P_{SN}(A)$; and the probability that if nothing is presented, the observer's input will lead to a "yes" response, $P_N(A)$. $P_{SN}(A)$ is the detection probability and $P_N(A)$ is the false-alarm probability.

In FIGURE 3,

$$P_{SN}(A) \approx \frac{n_1}{n_1 + n_2}$$

and

$$P_N(A) \approx \frac{n_3}{n_3 + n_4}$$

where the n's represent a counting of the various joint events occurring during the experiment.

The fact that we summarize the data as estimates of probabilities implies that the tasks the observer is asked to perform in psychophysical experiments are tasks in which there is a probability of error; consequently they satisfy Middleton's criterion (personal communication) as tasks that can be handled

within the framework of decision theory. According to this criterion, whenever one is confronted with a probability of error he is confronted with a decision theory problem.

Decision Theory Model

Decision theory serves as a foundation for the theory of signal detectability, a theory that deals with exactly the problem with which the observer is confronted in a psychophysical experiment. An examination of the general theory of signal detectability may help us understand the observer's problem.

The problem handled in this theory is that in which an observer is asked to state which of two conditions led to a particular input, x, during a fixed observation interval, 0 to T. Any input x is considered to be a function of time, $x(t)$, since time is the dimension of the observation interval. With each $x(t)$ two numbers are associated: the likelihood of that particular $x(t)$, if the signal was transmitted, $f_{SN}[x(t)]$; and the likelihood of that particular $x(t)$, if nothing was transmitted, $f_N[x(t)]$. The decision theory problem is to select a criterion space, A, which includes all of those $x(t)$s that should lead to the response "yes" and excludes all of those $x(t)$s that should lead to the response "no."

This can be stated as the problem of choosing the criterion space A such that

$$P_{SN}(A) - WP_N(A) = a \max \tag{1}$$

where W is a number or weighting function.

The following proof, derived by Fox,[1] demonstrates that the likelihood ratio describes the information contained in the input, $x(t)$, relevant to the choice the observer must make. Equation **1** can be written as

$$\int_A f_{SN}[x(t)]\, dx(t) - W\int_A f_N[x(t)]\, dx(t) = a \max.$$

Since both variables are over the same space, this becomes

$$\int_A f_{SN}[x(t)]\, d - Wf_N[x(t)]\, dx(t) = a \max.$$

In this form it becomes obvious that the integral is a maximum if all positive values are included and no negative values are included. Thus the space, A, should include all of those values of $x(t)$, and only those values, which satisfy the following inequality:

$$f_{SN}[x(t)] - Wf_N[x(t)] \geq 0$$

or

$$l[x(t)] = \frac{f_{SN}[x(t)]}{f_N[x(t)]} \geq W$$

where $l[x(t)]$ is defined as the likelihood ratio.

What is an Ideal Observer?

Given Fox's proof, we are now in a position to define an ideal observer, illustrated in its general form in FIGURE 4. It is an observer who computes the likelihood ratio associated with the input, $x(t)$, and then matches this value to a point in the criterion space. If the point is contained in A, then it says "yes," the signal was transmitted. If it is not in A, then it is in the complement of A, CA; and the observer says "no," the signal was not transmitted. The computation of $l[x(t)]$ and the matching are performed without error.

If we were to attempt to designate in detail the construction of an ideal observer, it would be necessary to know the signals that might be transmitted,

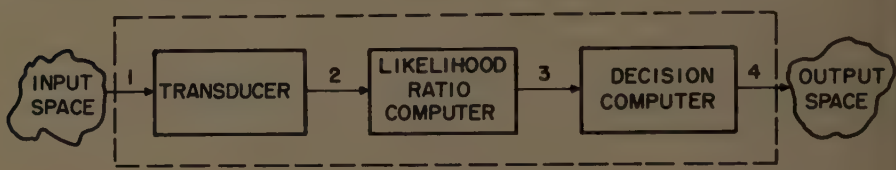

FIGURE 4. Ideal observer.

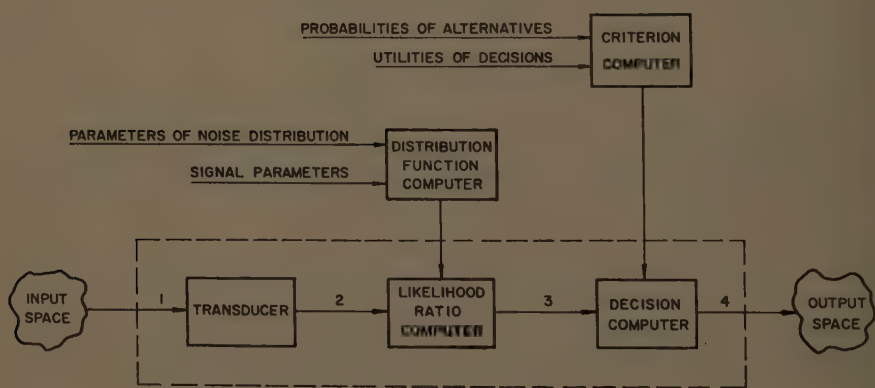

FIGURE 5. The ideal observer with tie-ins to the system.

the properties of the perturbing noise, the way in which the signal and noise are combined, and the function W, to be maximized. All of these parameters are required to specify the specific components (such as filters and integrators), and the particular criterion spaces, A and CA. Any change in the values of parameters can lead to a change in the detailed designation of the ideal observer. Thus if we want to consider the ideal observer as applied to specific situations, then there is an ideal observer for each specific case. They are all alike in that they compute likelihood ratios and say "yes" if the likelihood is greater than W. Whether any specific $x(t)$ leads to a "yes" or a "no" response depends on the signal and noise conditions from which it might have arisen and on the particular function, W, used in the maximization.

The whole operation is illustrated in FIGURE 5. The part of the block diagram included in the dotted lines can be considered as an ideal sensory system.

Those parts outside the dotted lines are the connecting links between the sensory system and the larger system that the sensory system serves. The connecting links mold the sensory system into an ideal form for the particular task at hand. It sets the parameters of the components and specifies the criterion spaces in a way that leads to optimum performance.

The Human Observer and the Ideal Observer

We probably all agree that, in the strict sense of the word, the human observer cannot be considered as an ideal observer. Few of us would expect perfect computations of likelihood ratios and perfect matching into the criterion spaces.

However, we might feel that the human observer's sensory system is, in a broad sense, similar to that of an ideal observer in some respects. It is, after all, a subsystem of an "intelligent" system and it seems not unlikely that the intelligent system would have some control over the form the subsystem assumes in facing any particular task. It may be molded in a way such that it exhibits a phenomenon we frequently refer to as "attention." Its performance may vary in a way that is consistent with different methods for computing W. While it may not achieve an ideal performance, it may in some ways tend toward an ideal performance.

If this is the case, then the experimenter must interpret his results in terms of the particular experimental conditions that have the potential of leading to a modification of the system. If these modifications are possible, then the physiologist is faced with a severe task in attempting to interpret data collected in experiments that eliminate the effects of the modifying agents. This line of reasoning clearly indicates physiological studies employing "consciously" behaving animals with implanted electrodes: the type of study I have heard Walter Rosenblith advocate many times.

FIGURE 6 shows a block diagram of an observer based on the ideal observer, but it also shows some of the places in which one might expect imperfections to exist. This diagram has as its basic structure that of the ideal observer, and the same links connecting the observer to the larger system. It has had added to it a number of memory banks that are immediately suggestive of imperfections. Let us examine the various properties an ideal memory must have in a simple detection experiment where the signal is a segment of a sine wave. This signal is specified exactly; thus when it occurs in the observation interval, 0 to T, it will occur at a precisely specified position within the interval. Its frequency, phase, and amplitude will also be specified precisely. This means that the function of time representing the signal can be positioned exactly in the observation interval. The value of this function can be precisely stated for any moment of time in the observation interval.

To be able to do this, an observer must have stored in his memory a precise representation of the function of time. He needs a clock mechanism that permits him to synchronize exactly with the observation interval, 0 to T, and to locate exactly in the interval the point at which the function of time should start, and precisely its duration. His memory must specify exactly the frequency, phase, amplitude, starting time, and duration of the segment of the sine wave. Any error in the memory will lead to a decrement in performance.

It is possible to make some inferences from psychophysical experiments regarding the role that each of these aspects of memory plays in the human observer's performance. It must be remembered, however, that these inferences are based on a model designed to describe the relation between the observables in a psychophysical experiment: the transmitted signals and the responses. The models can be looked at as mathematical equations fitting the data. As long as the data are finite, there are an infinite number of models that will fit equally well. Data collected in the future may eliminate this model, but will never confirm it. It uses words such as "memory" for the parameters because these words have intuitive appeal. The physiologist

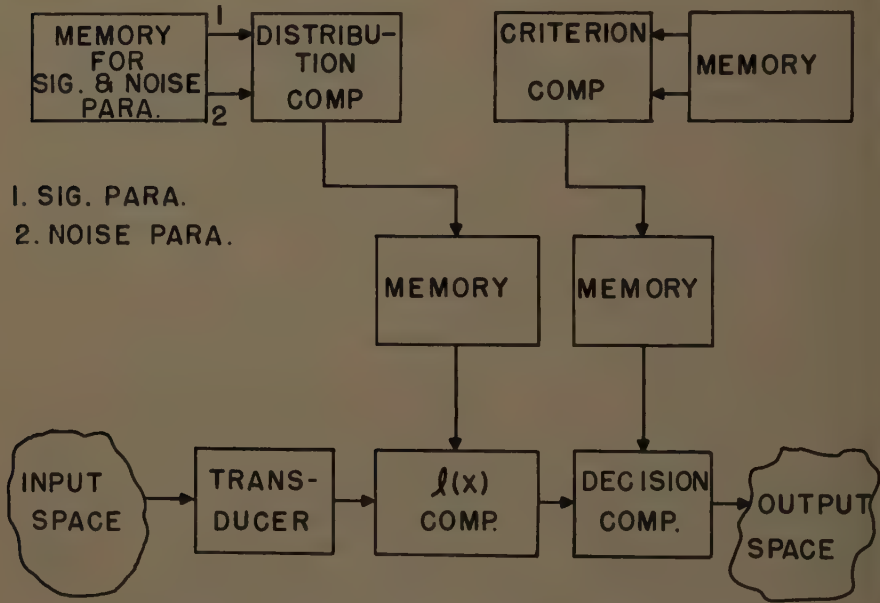

FIGURE 6. The ideal observer modified to include memory.

trying to correlate his observations with the parameters of the model must remember that the concept of memory arises from the model, not the data.

A Psychophysical Investigation of "Memory"

One of the concepts involved in using the theory of signal detectability as a tool for interpreting psychophysical data is expressed in the following "theorem":

If in two channels, one employing an ideal transmitter and an imperfect observer; and the other a transmitter with statistical uncertainty and an ideal receiver, both transmitters having equal energies available, the performance as indicated by measures of the false-alarm rate and the detection rate are found to be identical, then the imperfect receiver can be said to introduce the same degree of uncertainty in its channel as the transmitter in the other channel.

The uncertainty attributed to the receiver, it was felt, would reflect the observer's memory. Consequently a series of models with specified transmitter uncertainties were studied, leading to calculations of the performance measure d' as a function of $2E/N_0$. FIGURE 7 shows these plotted on log-log paper, and a human observer's data superimposed. The notion was that if the hu-

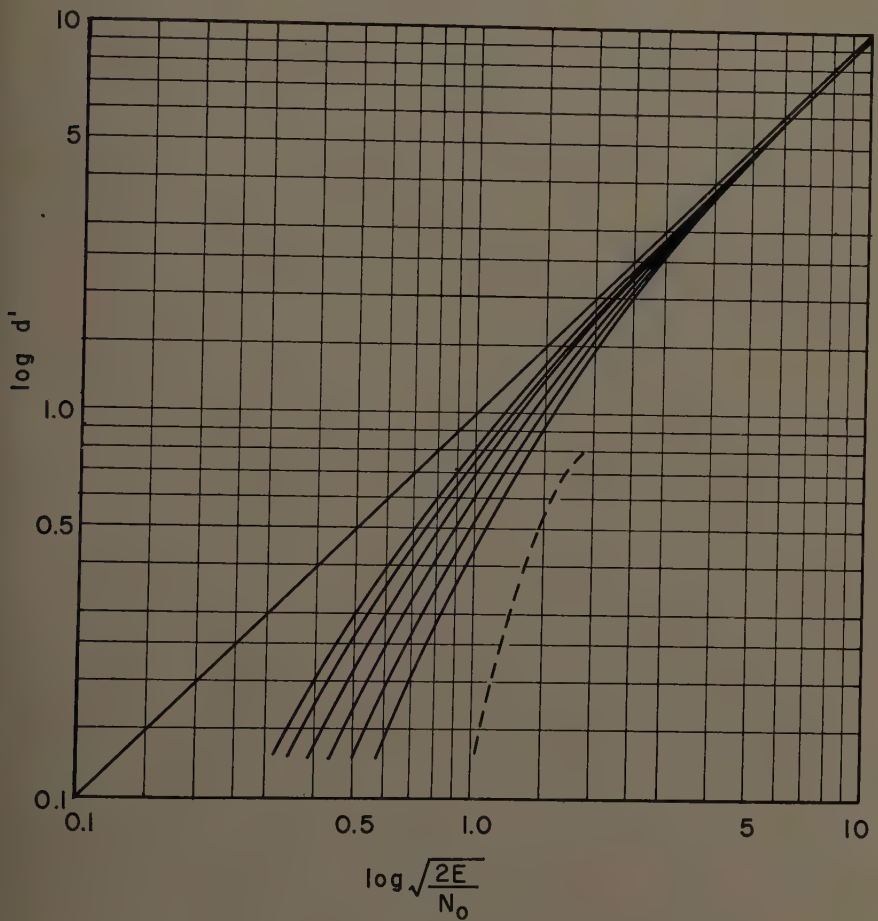

FIGURE 7. Performance curves for signals specified statistically.

man observer's data matched one of the curves, the degree of uncertainty introduced by him could be specified, and his memory characterized by a number.

The concept is far too simple. The human observer's data did not fall on any one curve, but rather cut across curves. This suggested that a different model would be required for every signal level, at least in terms of the number categorizing the memory.

Careful examination of the experimental procedures under which the data were collected suggests that this is not unreasonable. Each signal level was

studied in a sequence of 100 observations coming about five seconds apart. Before the sequence, the observer was permitted to listen to several samples of the signal without noise, and these noiseless samples were assumed to establish his memory.

If the signals are weak, then, as the sequence proceeds, the observer will receive few samples that are clear enough to reinforce his memory. If they are strong, he is more likely to receive some fairly good samples, and his memory on the average should be better throughout the sequence than it would be for the weak signals.

Elizabeth Shipley (personal communication) has recently performed an analysis of some data in which she considers the probability of a correct choice, given that the previous choice was correct and given that the previous choice was incorrect. She finds that a dependency exists when the signals are segments of sine waves, and does not exist when the signals are samples of noise. The dependency is in the right direction to support the hypothesis that a clear sample tends to reinforce the memory.

When Julian Bigelow saw some of the early results, he suggested a way of attacking the problem experimentally. He suggested that a sine wave of the same phase and frequency as the signal be added as a component of the noise, replacing the need for the observer's memory of frequency and phase. The signal would be superimposed on the sine wave during the observation interval. This procedure altered the slope of the psychophysical function so that it agreed with one of the slopes of the computed curves of FIGURE 7. However, it is positioned wrong.

There was still a need for the observer to have a memory for starting time and duration. My colleagues and I were able to extend Bigelow's suggestion in a way that could remove the need for this memory. This we have referred to as a "pulsed carrier" experiment. In each observation interval we introduced a segment of sine wave whose amplitude was large. This segment matched the signal in frequency, phase, starting time, and duration. It was the observer's task to state whether the signal appeared on top of this pulsed-carrier or pedestal, as J. C. R. Licklider calls it. In this case, the psychometric function turns out to be linear, or as if there were some added noise.

This leads to an interpretation of the memories required as a function of the experimental design, as illustrated in TABLE 1. There are three types of experiments: detection, continuous wave, and pulsed-carrier; each with two subdivisions, "yes-no" and 2AFC. The detection experiment requires a memory for frequency, phase, starting time, and duration. The CW experiment requires only starting time and duration memories, while the pulsed-carrier experiment requires none of these. The difference between a "yes-no" experiment and a 2AFC experiment is the need for amplitude memory in the "yes-no" experiment.

A summary of the results of the various experiments follows:

(1) In the detection experiment, the slope of the psychometric function is too great to be accounted for in terms of a fixed uncertainty. There is no difference between a "yes-no" and a 2AFC experiment, probably because the other memories are so poor that they mask the effect of amplitude memory.

(2) In the CW experiment, the slope of the psychometric function is such that it can be accounted for in terms of a fixed uncertainty. The function is misplaced, prohibiting an estimate of the degree of uncertainty. There is little, if any, difference between "yes-no" and 2AFC experiments.

(3) In the PC experiments, d' is a linear function $(2E/N_0)^{1/2}$, a condition that can be accounted for by additive noise. Performance in 2AFC experiments is noticeably superior to that in "yes-no" experiments.

The analysis of the memory conditions shown in TABLE 1 is not precisely true for an imperfect observer, who for example must have some amplitude memory in the 2AFC, PC experiment and must be able to remember the amplitude of the waveform in the first interval so that he can compare it to the amplitude of that in the second interval. In fact, the experiment can be looked at as a "yes-no" experiment in which the measure established in the first interval serves as a cut-off for the second. If the intervals are widely

TABLE 1
MEMORIES REQUIRED FOR DIFFERENT PSYCHOPHYSICAL EXPERIMENTS

Experiments		Required memories				
		Amplitude	Frequency	Phase	Starting time	Duration
Detection	"Yes-No"	+	+	+	+	+
	2 AFC		+	+	+	+
Continuous wave	"Yes-No"	+			+	+
	2 AFC				+	+
Pulsed carrier	"Yes-No"	+				
	2 AFC					

separated, this cut-off would not be precisely remembered. It might be as if noise were added to this cut-off, the amount of noise depending directly on the time between the two pulses.

Cut-off variance can be treated as additive noise, provided that all of the prior stages are linear, as illustrated in FIGURE 8. There are three distributions shown: in each the abscissa is the value of the decision variable, and the ordinate is the probability density of that variable. The distribution to the left is for the second signal, given that it is the smaller of the two. The distribution to the right is for the second signal, given that it is the larger. The middle distribution is the critical value of the decision variable, x_c.

It is possible to transform to a new decision axis, $x - x_c$, where on this new axis there is a fixed cut-off at 0. There are now only two distributions, $x - x_c$ given the smaller signal, and $x - x_c$ given the larger signal. Each of these has added the variance of x_c to its original variance.

Using the form of the psychometric function for the pulsed-carrier to demonstrate that the stages prior to the cut-off are linear, it should now be possible to study the variance of the cut-off as a function of the time, T, between the observation intervals. One complication still exists, however: there must

be a transducer type of operation at the input and, if T is too small, independent observations cannot be expected. Thus the performance should drop both for very small values of T and for very large values of T. In between there should be a peak. In fact, the following equation should specify d':

$$(d')^2 = \eta_0(1 - e^{-eT}) \frac{2E_\Delta}{N_0 + kT}$$

where e is something like the time constant of the transducer, k is the noise energy per unit time added to the cut-off value, and η_0 represents the efficiency if added noise of the nervous system alone accounts for the fact that the efficiency is less than unity. A typical set of results is shown in FIGURE 9.

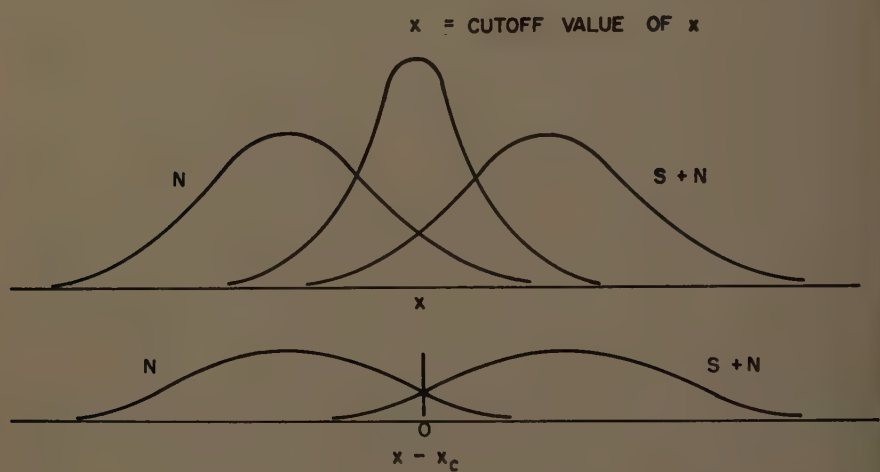

FIGURE 8. Illustration of the cut-off variance as added noise.

The CW experiment, using the same observer, may be re-examined now. Suppose it is assumed that the value of $2E/N_0$ used in the experiment has been reduced to the value of $(d')^2$, as calculated by the equation with the parameters determined by the pulsed-carrier experiment. One may now plot the psychometric functions as $(d')^2$ versus the corrected value of $2E/N_0$ (FIGURE 10). The misplacement of the curve is now corrected so that it is possible to estimate the degree of statistical uncertainty introduced by faulty memories of starting time and duration. The values of this parameter for the four observers range from about 5 to approximately 12.

All of the experiments thus far discussed have involved acoustic signals. My colleagues and I have also made visual studies, and R. Fitzhugh[2] at the Johns Hopkins University, Baltimore, Md., has reported some physiological data related to these studies. At first appearance there seems to be a discrepancy between Fitzhugh's data and ours. However, the discrepancy disappears when it is considered in terms of the memory framework.

Fitzhugh studied the frequency of nerve impulses on the dark-adapted cat's eye, counted over an arbitrary time interval after a flash of light had been

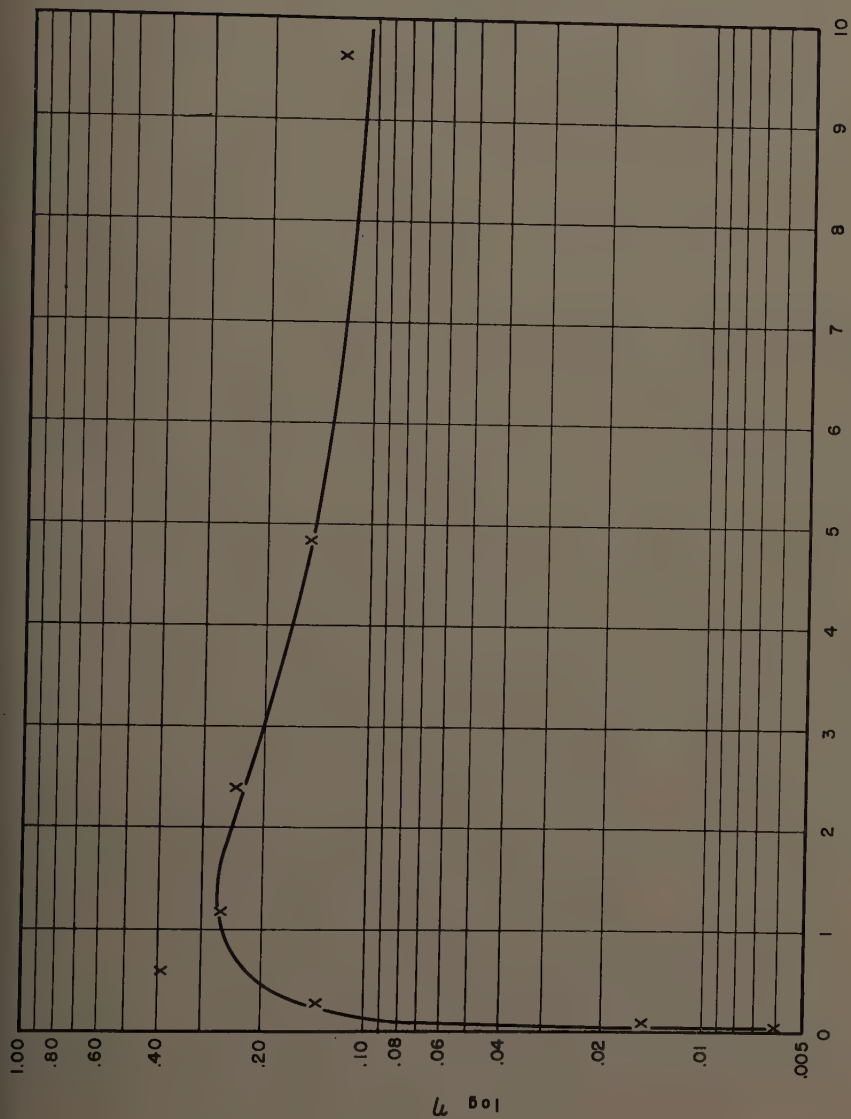

FIGURE 9. Efficiency as a function of amplitude memory time.

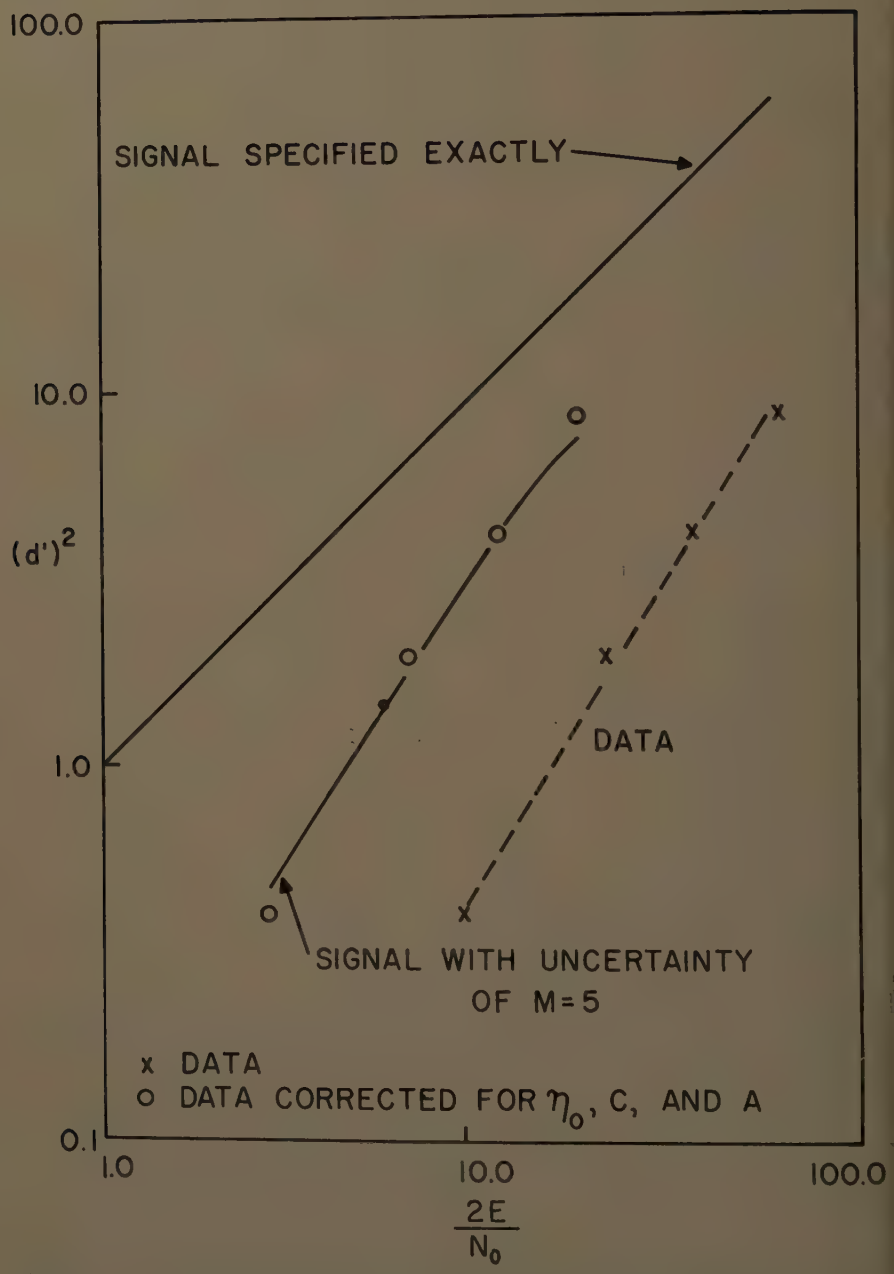

FIGURE 10 Data corrected to permit an estimate of starting time and duration uncertainty.

introduced. From his studies he was able to construct statistical distributions associated with each of a number of intensities (including 0 or noise alone). He could then assign values of d' representing the separation between any one of these distributions and that for noise alone. According to Fitzhugh, d' was approximately linearly related to ΔI, while in our psychophysical experiments d' varied as ΔI^p, where p was 2 or greater. An examination of our procedure indicates that there might be uncertainty both as to the time and location of the signal presentation. The slope of our curves agreed with the slope of the curves for $M = 100$. The uncertainty may be introduced at a central rather than retinal level, suggesting that one should not try to account for nonlinearities at the level of the end-organ, to the exclusion of central hypotheses.

Qne more point on the relation between Fitzhugh's results and ours. We analyzed our results on the assumption that normal distributions existed. Fitzhugh's distributions started at zero, and appeared quite skewed. This is not a discrepancy, for, from the standpoint of psychophysical analysis, any distribution that can be made normal by means of a monotonic transformation of the decision variable can be analyzed in a fashion consistent with the normal assumption.

To demonstrate this point, let us consider Fox's proof, which shows that likelihood ratio is the proper decision variable. Likelihood ratio varies from 0 to infinity and, for noise alone, at least, is highly skewed. However, if one considers the decision rule "say yes" if $\log_e l(x) \geq \log_e W$, every decision is the same as if it were made according to the decision rule "say yes" if $l(x) \geq W$. $\text{Log}_e\, l(x)$ varies from $-\infty$ to $+\infty$, and under certain conditions is normal. Since all the decisions are the same, and since one can only observe the decisions, a normal analysis can be applied whether the decision variable is $\log l(x)$ or $l(x)$. The results will be the same in both cases.

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DECISION MAKING AND LEARNING UNDER VARYING CONDITIONS OF REINFORCEMENT*

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From the work of philosophers, mathematicians, economists, and psychologists, a theory of rational choice has emerged that considers the nature of rational behavior in human decision-making.[1-10]

According to this approach, when a person can specify a set of alternative outcomes such that for any strategy p_i he knows the probability distribution of outcomes, that person is "rational" if he chooses a strategy that will enable *maximization of expected utility.* Most decision-making theorists take "probability" to mean subjective probability, and "utility" to mean subjective value. Thus a strategy p_i is admissible if there is no other strategy p_j such that

$$SEU\ (p_j) > SEU\ (p_i)$$

where $SEU\ (p)$ is the subjectively expected utility when strategy p is used by the person.

Compared to the considerable amount of sophisticated theoretical work that has been done in connection with this theory of rational choice, there has been relatively little empirical work. However, some empirical work articulated directly with the theory has been attempted.[7,10-13] In addition, there is a considerable body of empirical work by psychologists that can be interpreted in terms of the implications of the theory of rational choice.[14-24] However, when that theory is tested against the data presented in these studies, conflicting results are found. Whereas some of the studies support the predictions derived from a decision-making approach based on the theory of rational choice, others fail to support these predictions.

The principal purpose of this paper is to bring psychological approaches to human choice behavior into more direct articulation with the theory of rational choice already stated. The balance of the paper is concerned with a class of psychological experiments involving choice behavior and learning: the two-choice uncertain outcome situation. Mathematical models and experimental tests of these models are presented. These will attempt to integrate the psychological approach with the theory of decision making.

Stable-State (Asymptotic) Behavior in the Two-Choice Situation

We are concerned here with an experimental situation that has been widely used in studies of human learning and choice behavior, a variation of which was initially employed by Lloyd Humphreys.[25] The subject is seated before two lights and asked to predict which of the two will illuminate after a signal stimulus. On every trial, one and only one light illuminates. The two events occur with fixed but unequal probabilities, π and $1 - \pi$, in a random sequence

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for a number of trials. In some studies, devices other than lights have been used. In any case, the events are independent of the subject's behavior: regardless of what he does, one of the two predetermined events will occur on each trial. The subject is instructed to do his best to predict correctly which of the two will occur. He expresses his prediction by pressing one of two buttons. He is given no other verbal information about the situation, but he is allowed to witness the events and thus to determine for himself the correctness or incorrectness of each of his predictions.

W. K. Estes has developed a statistical theory of learning[14-16] that yields predictions about an individual's stable-state behavior in a situation of this sort. Estes' model yields the prediction that subjects will learn to match their response ratios (the relative frequencies with which they predict each of the two events) to the actual probabilities of occurrence of the events π and $1 - \pi$. The same prediction is given by the Bush-Mosteller model[23] when certain restrictions are applied to the parameters. A number of studies report findings in support of this prediction.[18-21]

This is a curious result. Since the subject is instructed to do his best to predict correctly which of the two events will occur, and since we may suppose that he is attempting to follow those instructions, should we not expect him to learn to maximize the expected frequency of correct predictions? To do this, he should tend to predict the more frequent event on all trials. We might expect him to come to such stable-state behavior after an initial learning period.

Consider, for example, a two-choice uncertain outcome situation in which over a series of trials the two events, E_1 and E_2, occur with probabilities π and $1 - \pi$ of 0.75 and 0.25 respectively. Estes' model asserts that a person will learn to make predictions that tend toward stabilizing at prediction of E_1 on 75 per cent of the trials ($p = 0.75$) and prediction of E_2 on 25 per cent of the trials. A person who adopts this "strategy" will have 0.625 as his expected proportion of correct predictions, for

$$\begin{aligned} E_x &= p\pi + (1 - p)(1 - \pi) \\ &= 0.75(.75) + 0.25(.25) = 0.625 \end{aligned}$$

On the other hand, if the subject were to adopt the "pure strategy" of predicting E_1 on every trial ($p = 1.0$), he would have 0.75 as his expected proportion of correct predictions, for

$$E_x = 1.0(.75) + 0(.25) = 0.75$$

Because of the considerations sketched above, some behavioral scientists have asserted that people who match their response ratios to the probabilities of the events are acting "irrationally" in that they are failing to maximize their expected proportion of correct predictions, a goal they could accomplish by predicting the more frequent event on every trial. The empirical fact is that in this situation most people do, after many trials, stabilize at matching their response ratios to the probabilities of E_1 and E_2. To assert that this is "irrational" is to rely on a highly restrictive meaning of that term. It was pointed out by Bernoulli[1] in the first half of the 18th century that any theory

of rational behavior that does not incorporate the concept of the utility or *subjective* value of the outcomes rather than their *objective* value will lead to paradoxes of the kind under discussion. One must bear in mind the distinction between objective rationality (rationality as viewed by the experimenter) and subjective rationality (behavior that is rational, given the perceptual and evaluational premises of the subject).

The purpose of the research to be presented here is to suggest, and demonstrate the validity of, a resolution of the apparent paradox. In this research a utility theory approach is incorporated.

From decision-making theory[6,7] a hypothesis of maximization of expected utility may be drawn that will account for both sorts of prediction behaviors: both matching response ratios and maximizing the expected frequency of correct predictions. According to this approach, which has been formalized in two models to be presented later in this paper, whether a person will tend toward one or the other prediction strategy depends on certain conditions related to the reinforcement inherent in the situation. The general hypothesis is that a person will behave as if he were attempting to maximize expected utility in any instance. When the components of the total available utility vary in magnitude from situation to situation, we may expect a person's strategy to maximize expected utility to vary concomitantly.

It seems reasonable to suppose that when a person is in a situation in which the only payoff or reinforcement from the outcomes is the satisfaction of seeing his prediction confirmed by the event or the dissatisfaction of seeing it disconfirmed, making a correct prediction of the *rarer* event has greater utility for the person than making a correct prediction of the more frequent event. The person derives satisfaction from "playing a game with the machine," trying to "outwit" it.

Moreover, there is the matter of monotony, both kinesthetic and cognitive. Predicting the more frequent event on all trials, that is, utilizing the maximizing strategy, would engender the monotony of pressing the same button on trial after trial for hundreds of trials. It would also engender the monotony of experiencing the same cognitive response, for example, "left button," on trial after trial.

Under such circumstances, a subject may maximize expected utility by using a mixed strategy of splitting his predictions in some proportion. Matching his response ratio to the actual probabilities of occurrence of the two events would be one possible mixed strategy. The "cost" of an incorrect prediction is very low, whereas the gain this strategy may yield in terms of other utilities—such as the utility of gambling and the utility of variability (relieving monotony)—may be relatively high. Thus a mixed strategy may serve to maximize total satisfaction.

If such an account is correct, then a decision-making model could yield the same predictions as the Estes model concerning the behavior of a person in a two-choice situation involving no special payoff.

However, if the utility attached to correct predictions were somehow increased by a change in the conditions of the game, then the prediction yielded by the hypothesis of maximization of expected utility would diverge from that yielded by the Estes model. One may change the game so that there is poten-

tial satisfaction beyond that of having an event confirm one's prediction, or so that the cost of an incorrect prediction is increased. If the decision-making theory approach proposed here is correct, the introduction of systematic variation in the reinforcements (and thus utilities) attached to correct or incorrect predictions should be followed by systematic differences in subjects' responses. As the utility of a correct prediction is increased and/or as the cost (negative utility) of an incorrect prediction is increased, the subject's prediction of the more frequent event should tend to 100 per cent. That is, he may be expected to choose a "pure strategy" rather than a "mixed" one.

In the section that follows a formulation of the decision-making theory approach is given that is more precise than the informal comments above. This formulation, Model I, centers on two concepts: the utility of a correct prediction and the utility of variability. In Model I there is no explicit consideration of the utility of gambling. Thus, there is an implicit assumption—and this may be unreasonable—that when data on decision making consist of averages for groups of subjects there is little, if any, systematic effect from the utility of gambling, since this utility is positive for some people and negative for others.

Model I

Let π = the probability of occurrence of the more frequent event;
p = the proportion of times the subject chooses the more frequent event;
a = the marginal utility of a correct prediction; and
b = the marginal utility of variability.

If the expectation that a subject's prediction will be correct, E_x, is

$$E_x = [p\pi + (1 - p)(1 - \pi)]$$
$$= [(1 - \pi) + p(2\pi - 1)]$$

then the expected utility of a correct prediction, U_r, is

$$U_r = aE_x = a[(1 - \pi) + p(2\pi - 1)]$$

If U_v = the utility of variability = $f(p)$, such that U_v is symmetrical and maximal for $p = 0.5$, for example, $p(1 - p)$, then

$$U_v = bp(1 - p)$$

The total expected utility of a particular strategy p is

$$U(p) = U_r + U_v$$
$$= a[(1 - \pi) + p(2\pi - 1)] + bp(1 - p)$$

The strategy p that maximizes expected utility is at

$$\frac{dU(p)}{dp} = 0$$

and is

$$p = \frac{a(2\pi - 1)}{2b} + \frac{1}{2}$$

If $\alpha = (a/b)$
then

$$p = \alpha(\pi - \tfrac{1}{2}) + \tfrac{1}{2} \tag{1}$$

It may be seen from **1** that when and only when $a = b$, and thus $\alpha = 1$, $p = \pi$. That is, when the marginal utility of a correct prediction equals the marginal utility of variability, Model I makes the same prediction regarding asymptotes as the Estes model. This situation seems to hold when the only reward in the situation is knowing whether one's prediction is correct or not.

However, if the reinforcements inherent in the situation are increased, for example by adding monetary rewards and/or costs to the outcomes, and thus the utility of a correct prediction and/or the negative utility of an incorrect prediction is increased, then $a > b$ and $\alpha > 1$. For such conditions the stable-state predictions yielded by Model I diverge from the predictions yielded by the Estes model.

The predictions Model I yields concerning a subject's choice and strategy behavior may be generally stated as follows: For any π

(1) $p = 1$ when $\alpha \geq [1/(2\pi - 1)]$

(2) $1 > p > \pi$ when $1 < \alpha < [1/(2\pi - 1)]$

(3) $p = \pi$ when $\alpha = 1$

(4) $p < \pi$ when $\alpha < 1$

The third prediction, that $p = \pi$, is the prediction yielded by the Estes model, and is a special case of Model I. The first, second, and fourth predictions, it would seem, the Estes model is not prepared to make.

Experimental Tests of Model I

To date we have completed three independent experimental tests of Model I. In each of these experiments, the subjects were male undergraduate students and the experimenter was a male psychologist. Each subject was seen individually in a personal experimental session. He was seated before a board to which a right and a left light plus a signal light were attached. He was instructed that his task was to predict whether the right or the left bulb would illuminate after the signal stimulus. He expressed his prediction by pressing a button. Then he would observe which light in fact did illuminate. The two lights illuminated according to a previously fixed random series, one illuminating on π proportion of the trials, the other on $1 - \pi$ proportion. To control for possible position preferences, we used the right and the left lights as the more frequent ones for different subjects, at random, with 50 per cent of the subjects participating in sessions in which the left light was the more frequent one and 50 per cent participating in sessions in which the right light was the more frequent. The operation of the lights was controlled by electronic equipment, and the subject's choices were recorded automatically. The experi-

menter was seated in the room in such a location that it was evident to the subject that the experimenter was not actively operating or tinkering with the board, nor was he in any way determining which light illuminated on any trial.

Each subject was exposed to a large number of trials (200 to 300) to give him an opportunity to learn the situation and to reach a stable-state strategy. Our special interest in these three studies was in the asymptotic level of p (proportion of predictions of the more frequent light) that a subject would reach after many trials.

In the various experiments we used up to three conditions of reinforcement in an attempt to manipulate the utility attached to a correct prediction and the negative utility attached to an incorrect one. The three conditions were: No Payoff, Reward, and Risk.

Under the *No Payoff* condition, the reinforcement for each prediction consisted simply of seeing the outcome—whether the right or left bulb lit—and thereby determining whether one's prediction was correct. This condition is the familiar and usual one in studies of human behavior in two-choice situations. Under the *Reward* condition, the subject received five cents for each correct prediction, the nickel being given to him immediately after each trial on which his prediction was confirmed. Under the *Risk* condition, the subject received a nickel each time his prediction was confirmed and forfeited a nickel if it were disconfirmed. Thus, under the Reward and Risk conditions, there was monetary reinforcement in addition to the reinforcement involved in determining the correctness of one's prediction. The two differed in that the Reward condition involved only increasing the utility of a correct prediction, whereas the Risk condition involved increasing the negative utility of an incorrect prediction as well.

Every subject, regardless of condition, was given a stake at the start of the experimental session, and told that whatever money he held at the conclusion of the session would be his to keep.

First test of Model I. In the first experiment[26] 36 men served as subjects, and they were randomized into three groups. These were run under the three conditions already described: No Payoff, Reward, and Risk. For this experiment, π was set at 0.75. That is, one light illuminated on 75 per cent of the trials and the other illuminated on the remaining 25 per cent. The order of occurrence of the two events was randomized into blocks of 20 trials, and subjects were run for a total of 300 trials. Every subject was given a stake of 75 cents at the start of his session.

The purpose of the experiment was to test a hypothesis drawn from the decision-making approach already introduced, a hypothesis formalized in Model I. Since the predicted strategy is

$$p = \alpha(\pi - \tfrac{1}{2}) + \tfrac{1}{2} \tag{1}$$

where $\alpha = a/b$, it was hypothesized that an increase in the value of a, achieved by offering an additional reward for correct predictions, would lead to an increase in p.

That is, using Estes notation,[15] where $\bar{p}_1(\infty)$ is the mean asymptotic probability of predicting E_1 for a group of like individuals,

$$\text{under no payoff, } \bar{p}_1(\infty) = \pi_1$$
$$\text{under reward, } \bar{p}_1(\infty) = \pi_1 + \epsilon$$
$$\text{under risk, } \bar{p}_1(\infty) = \pi_1 + \epsilon + \lambda$$

The hypothesis, then, was that

$$\bar{p}_1(\infty), \text{ under risk} > \bar{p}_1(\infty), \text{ under reward} > \bar{p}_1(\infty), \text{ under no payoff.}$$

The results confirmed the prediction at a highly significant level ($p < 0.001$). We determined the proportion of times that each subject predicted the more frequent light during the final 20 trials of his series of predictions. On the final 20 trials of the first 100, these proportions were

$$\bar{p} \text{ (No Payoff)} = 0.69$$
$$\bar{p} \text{ (Reward)} = 0.78$$
$$\bar{p} \text{ (Risk)} = 0.90$$

By the end of 300 consecutive trials, when we may more safely speak of choices on the final 20 trials as representing asymptotes, the data were

$$\bar{p} \text{ (No Payoff)} = 0.75$$
$$\bar{p} \text{ (Reward)} = 0.86$$
$$\bar{p} \text{ (Risk)} = 0.95$$

That is, as predicted, subjects under the Risk condition will predict the more frequent light oftener than subjects under the Reward condition, and these in turn will predict the more frequent light oftener than subjects under the No Payoff condition. Thus the data lend strong support to the contention from decision-making theory that a person behaves in this situation as if he were attempting to maximize expected utility. The utility hinging on correct or incorrect predictions differed under the No Payoff, Reward, and Risk conditions, and thus it seems that the attempt to maximize expected utility led to different strategies under these conditions.

Second test of Model I. The first test offered strong support for the general features of Model I. However, evidence was still needed to support its particular features. That is, although the data supported the ordinal hypothesis that an increase in the marginal utility of a correct prediction, a, leads to an increase in p, the proportion of time the more frequent light is predicted, this finding cannot be taken as support for such particular features of Model I as, for example, that $U_v = bp(1 - p)$. This expression, although it has considerable intuitive appeal as an expression for the expected utility of variability is, after all, an arbitrary one. Many other expressions could be used that would be consonant with the ordinal hypothesis supported in the first experiment. It was to test the specific mathematical structure of Model I that the second experiment was designed.

For the second test[27] 40 subjects were randomized into two equal groups, and

both groups were run under the Risk condition. The first group was run with $\pi_1 = 0.75$, and from their data an $\hat{\alpha}$ was estimated. This value was used to predict the value of p_2 obtained from the second group of subjects. These were run with $\pi_2 = 0.65$. All subjects were run for 300 trials, and were given a stake of 75 cents at the start of the session.

Under Model I, $\hat{\alpha}$ is obtained from the data as follows:

$$p = \alpha(\pi - \tfrac{1}{2}) + \tfrac{1}{2} \tag{1}$$

and thus

$$\hat{\alpha} = \frac{p_1 - \frac{1}{2}}{\pi_1 - \frac{1}{2}}$$

The design of the study may be represented as

	$\pi_1 = 0.75$	$\pi_2 = 0.65$
risk condition	$\hat{\alpha}$	predict p_2

and the prediction was

$$p_2 = \hat{\alpha}(\pi_2 - \tfrac{1}{2}) + \tfrac{1}{2}$$

The results of this study offer strong support for the particular features of Model I. For the first group of 20 subjects run under Risk with $\pi_1 = 0.75$,

$$\bar{p}_1 = 0.929$$

and

$$\hat{\alpha} = 1.718,$$

and thus the predicted value of $\bar{p}_2$ was

$$p_2 = 1.718\ (0.65 - \tfrac{1}{2}) + \tfrac{1}{2} = 0.758$$

When the second group of subjects was run under $\pi_2 = 0.65$ and under the same level of Risk as the first group, the subjects reached an asymptote of 0.753 at the end of 300 trials. This value was remarkably close to the value predicted from Model I on the basis of knowledge of the value of $\hat{\alpha}$ in the first group:

$$\text{predicted value of } p_2 = 0.758$$

$$\text{observed value of } \bar{p}_2 = 0.753$$

In addition to supporting the specific character of Model I, these data also support the notion that subjects will not match response ratios to input ratios when correct predictions are given increased reinforcements. Thus for the first group the input probability was $\pi_1 = 0.75$, and the responses stabilized at $\bar{p}_1 = 0.93$. For the second group, the input probability was $\pi_2 = 0.65$ and the responses stabilized at $p_2 = 0.75$. These results are not unexpected in the light of the findings of the first study.

In a later study, an opportunity arose to replicate this second test of Model I. Twenty subjects were run under Risk at $\pi_3 = 0.70$. On the basis of the finding that $\hat{\alpha} = 1.718$ in the first study, we would predict for this third group that $p_3 = 0.84$. In fact, the responses stabilized at $\bar{p}_3 = 0.85$, a value in close agreement with the predicted value, and one that provides additional support for the specific characteristics of Model I.

Third test of Model I. The two studies summarized above support the general and specific features of Model I. That is, the data are evidence in support of the *mathematical* definitions of the variables of the model. However, they do not establish the validity of the *psychological* definitions of these hypothetical constructions.

The expression in Model I for the expected utility of a correct prediction, $U_r = aE_x$, has considerable face validity and a sound logical foundation. Thus it was deemed advisable to focus attention on attempting to validate the variable that is perhaps psychologically more interesting, the expected utility of variability, $U_v = bp(1 - p)$.

We may argue that if U_v really stands for the utility of varying one's responses, then by giving subjects an opportunity to make the same prediction without being required to experience the same kinesthetic and cognitive responses, we should reduce the marginal utility of variability, b, and thus raise $\alpha = a/b$. Since $p = \alpha(\pi - \frac{1}{2}) + \frac{1}{2}$, an operation that increases α would increase p, the proportion of predictions of the more frequent event. Thus it is predicted that

$$\bar{p} \text{ (low } b) > \bar{p} \text{ (No Payoff)}$$

The third test of Model I is based on the notion that the utility of variability can be reduced by giving the subject an opportunity to make the same prediction, for example "left light," by different responses. We devised an experimental situation that should operate to reduce the monotony engendered by making the same prediction on trial after trial by providing opportunities for cognitive and kinesthetic variability in making the same prediction.

In this study, each subject in the experimental (low b) group was seated in a swivel chair located between the two-light board and a large mirror. Each trial began with the subject facing at right angles to the board and to the mirror. The signal stimulus consisted of two arrows, one of which was lighted to start each trial. In a random sequence, one half of the time the arrow was lighted that directed the subject to make his prediction at the board, and the other half of the time the arrow was lighted that directed the subject to make his prediction at the mirror. There were two "prediction" buttons in front of the mirror as well as the usual two in front of the board. In this situation, a subject desiring to predict the illumination of the left light would press the left button when facing the board but the right button when facing the mirror since, in the latter case, he would see a "mirror image" of the board and make his prediction according to the position of the lights in this image. In addition to the cognitive variability involved in such a situation, there was considerable kinesthetic variability due to the necessity for him to swivel from the signal light position to the board (or mirror), back to the signal light for the start of

the next trial, thence to the mirror (or board), and so forth. In the situation, a subject could choose the more frequent stimulus event on all trials (adopt a pure strategy) while pressing a left button 50 per cent of the time and a right button 50 per cent of the time.

In this experiment[28] 60 subjects were randomized into two groups, one group run in the experimental (mirror) condition and one in the control condition, π being set at 0.75, and each subject run individually for 240 trials. Both groups were run under No Payoff. The control subjects were run, thus, in the classical two-light situation.

The results of this study provided support for the prediction that the average p of the experimental subjects would be higher than the average of the controls. The average proportion of predictions of the more frequent light by the control subjects was

$$\bar{p}_c = 0.74$$

whereas for the experimental group the average was

$$\bar{p}_e = 0.80$$

The difference between these means is significant at $p < 0.05$.

Summary and Conclusions

To date, three experiments have been conducted concerning various features of Model I.

The first experiment confirmed the general features of the model by demonstrating that the prediction strategies that subjects reach differ according to the reinforcement inherent in the situation. Under No Payoff, the asymptotic rate of predicting the more frequent event is lower than it is under Reward. In turn, the prediction rate under Reward is lower than it is under Risk.

The second experiment supported the specific mathematical structure of Model I. On the basis of a value of $\hat{\alpha}$ derived from the Model with data from one group of subjects performing under Risk with $\pi = 0.75$, a prediction was reached concerning the strategy a second group of subjects would stabilize at under the Risk condition with $\pi = 0.65$. The predicted value was $p_2 = 0.758$, and this was remarkably close to the observed value, $\bar{p}_2 = 0.753$.

The third experiment was directed toward validating the psychological meaning of one of the important expressions in the model, U_v, the expected utility of variability. Subjects were given an opportunity to vary the kinesthetic and cognitive experiences associated with their responses without varying the prediction represented by those responses. Under these circumstances, p increased, as predicted from Model I.

Thus all three experiments have supported hypotheses derived from Model I. It should be noted, however, that this is a relatively simple model, with few variables. It does not incorporate a method for dealing with—measuring or controlling—the specific utility of gambling. This term refers perhaps to such satisfactions as are inherent in "playing a game with the machine," "winning on a long shot," and "successfully predicting the rarer event." To handle this concept, which may well be important in determining subjects' prediction

strategies, a second model was developed that is more general and "richer" than Model I. This approach, Model II, can now be presented. At present we are beginning to conduct studies testing Model II, and a description of the experiments now in progress is included in the presentation of Model II that follows.

Model II

Let π = the probability of occurrence of the more frequent event, as in Model I;

p = the proportion of times the subject chooses the more frequent event, as in Model I;

a = the marginal utility of a correct prediction when and only when the subject chooses the more frequent event;

b = the marginal utility of a correct prediction when and only when the subject chooses the less frequent event; and

c = the marginal utility of variability.

The expected utility of a correct prediction, $E_x(U_r)$, is

$$E_x(U_r) = ap\pi + b(1 - p)(1 - \pi)$$

and the utility of variability, U_v, is $f(p)$, as in Model I, and is

$$U_v = cp(1 - p)$$

The total expected utility of a particular strategy p is

$$U(p) = ap\pi + b(1 - p)(1 - \pi) + cp(1 - p)$$

The strategy p that maximizes $U(p)$ is at

$$\frac{dU(p)}{dp} = 0$$

and is

$$p = \frac{a\pi - b(1 - \pi) + c}{2c}$$

If $\alpha = a/c$ and if $\beta = b/c$, then

$$p = \frac{(1 - \beta) + \pi(\alpha + \beta)}{2} \tag{2}$$

It may be seen from **2** that when $a = b = c$, and thus $\alpha = \beta = 1$, Model II yields, as a special case, the Estes prediction that $p = \pi$. It will be shown later that the case when $\alpha = \beta = 1$ is not the only case for which $p = \pi$ according to Model II.

Model II is richer than Model I in several respects. One is that it makes possible experimental studies involving differential payoffs for the more frequent and less frequent events. In addition Model II may lead to the measurement of an individual's specific utility of gambling.

The predictions Model II yields concerning a subject's choice and strategy behavior may be generally stated as follows: For any π,

(1) $p = 1$ when

$$\alpha \geq \frac{(1 - \pi)\beta + 1}{\pi}$$

and/or

$$\beta \leq \frac{\pi\alpha - 1}{1 - \pi}$$

(2) $1 > p > \pi$ when

$$\frac{(1 - \pi)\beta + 1}{\pi} > \alpha > \frac{(1 - \pi)\beta + (2\pi - 1)}{\pi}$$

and/or

$$\frac{\pi\alpha - 1}{1 - \pi} < \beta < \frac{\pi\alpha - (2\pi - 1)}{1 - \pi}$$

(3) $p = \pi$ when

$$\alpha = \frac{(1 - \pi)\beta + (2\pi - 1)}{\pi}$$

and/or

$$\beta = \frac{\pi\alpha - (2\pi - 1)}{1 - \pi}$$

(4) $p < \pi$ when

$$\alpha < \frac{(1 - \pi)\beta + (2\pi - 1)}{\pi}$$

and/or

$$\beta > \frac{\pi\alpha - (2\pi - 1)}{1 - \pi}$$

The first, second, and fourth predictions—that p will be 1, will be between 1 and π, or will be less than π—are predictions regarding asymptotes that the Estes model is not prepared to make. The third prediction—that p will be at π—is the Estes prediction, and occurs here as a special case (or set) in Model II.

In Model II, there are two parameters, α and β, which must be estimated before the value of p can be predicted:

$$p = \frac{(1 - \beta) + \pi(\alpha + \beta)}{2} \tag{2}$$

Therefore a study is needed in which two groups of subjects are run under different π's and one level of reinforcement, R_1. Data are required, that is, from a study of this design:

	π_1	π_2
R_1	p_1	p_2

With π_1 and π_2 set, and with p_1 and p_2 observed, according to Model II,

$$p_1 = \frac{(1 - \beta) + \pi_1(\alpha + \beta)}{2}$$

$$p_2 = \frac{(1 - \beta) + \pi_2(\alpha + \beta)}{2}$$

Solving these two equations simultaneously, we obtain estimates as follows:

$$\hat{\beta} = \frac{2(p_1\pi_2 - p_2\pi_1)}{\pi_1 - \pi_2} + 1$$

$$\hat{\alpha} = \frac{2(p_1 - p_2)}{\pi_1 - \pi_2} - \hat{\beta}$$

With $\hat{\beta}$ and $\hat{\alpha}$ computed, a study could then be conducted using a different π than those above, for example π_3, and, under the same reinforcement, R_1, as was used above, a test of Model II would be a test of the prediction.

$$p_3 = \frac{(1 - \beta) + \pi_3(\alpha + \beta)}{2}$$

An important difference between Model I and Model II is that Model II contains separate expressions for the utility of correctly predicting the more frequent event and the utility of correctly predicting the less frequent event. Thus whereas both models predict that p will depend on the reinforcement hinging on correct predictions, only Model II yields a prediction for the situation in which the reinforcement contingent on correctly predicting the more frequent event differs from the reinforcement contingent on correctly predicting the less frequent event. According to this model, by systematically varying these two types of reinforcement it should be possible to induce variations in p from zero to one!

Consider, for example, the Risk situation used above. The subject was confronted with two lights, one of which (for example the left one) illuminated on 75 per cent of the trials ($\pi = 0.75$) and one of which (the right) illuminated on 25 per cent ($1 - \pi = 0.25$). If the subject predicted that the left light would illuminate and it did, he received 5 cents. If it did not, he forfeited 5 cents. On the other hand, if he predicted that the right light would illuminate and it did, he received 5 cents, and he forfeited 5 cents if it did not. This reinforcement situation may be represented schematically as follows:

	left (0.75)	right (0.25)
prediction confirmed	+5 cents	+5 cents
prediction disconfirmed	−5 cents	−5 cents

Under such a reinforcement situation, it is clearly to the subject's monetary advantage (in terms of expected value) always to predict the illumination of the left light, and we presume that the reason subjects fail to follow this strategy is that they are attempting to maximize expected utility rather than expected value, and that the total utility inherent in the situation includes other components besides the monetary one—the utility of variability, for example.

Suppose, now, that the reinforcement situation were as follows:

	left (0.75)	right (0.25)
prediction confirmed	+5 cents	+5 cents
prediction disconfirmed	−5 cents	−$1.00

Under such a situation, the expected value associated with predicting the illumination of the left light is much higher, and we would expect the utility of money to practically override any competing utilities that induce occasional selection of the right light, and that p would be closer to unity in this situation than in the preceding one. Such a prediction is yielded by Model II.

Consider, on the other hand, this situation:

	left (0.75)	right (0.25)
prediction confirmed	+5 cents	+5 cents
prediction disconfirmed	−5 cents	+$1.00

Under this situation of reinforcement, we would expect p, the proportion of times the subject predicts the illumination of the left light, to be far lower than under either of the preceding situations, and in fact to be near or at zero.

The above examples illustrate that by varying one monetary value in the reinforcement situation, from $+d$ dollars through 0 to $-d$ dollars, we can induce variation in the subject's p from zero to one. Such induced variation in p, if observed, would constitute partial verification of Model II.

With an experimental set-up such as that outlined above, it should be possible to obtain a measure of a subject's specific utility of gambling. To obtain such a measure, it would be necessary to know the subject's utility of money, and this could be determined by the method developed by Davidson *et al.*[7] The experimenter could then determine how large a change in money (in one of the cells showing reinforcement, as shown above) is necessary to change the sub-

ject's strategy, p, from some stable value to unity This result, under the proper experimental conditions, should yield a measure, in monetary units, of the subject's specific utility of gambling.

A Decision-Making Theory of Learning

The principal purpose of the models that have been presented is to formalize the notions of decision-making theory and to use them to predict stable-state behavior in a binary choice situation. It has been shown that the mathematical models presented above can predict such asymptotes, and do so for varying conditions of reinforcement.

A shortcoming of these models is that they imply nothing about the course of learning. In contradistinction, the Estes model[14-16] and the Bush-Mosteller model[23] have the power to describe the entire course of learning in the two-choice situation, but they have the shortcoming that they fail to explain the effects of varying conditions of reinforcement.

I have been working with Robert Radlow, a National Science Foundation postdoctoral Fellow, on extending the decision-making models so that a more general class of behaviors can be predicted. The extension of the decision-making models to learning behavior is an attempt at unifying the two approaches just mentioned, in that it allows prediction of the entire course of learning and of asymptotic choice behavior for different conditions of reinforcement.

The essential idea of the decision-making theory of learning is to regard an individual's choice on any given trial as the result of his degree of belief (subjective probability) that a given event will occur as well as the result of the utility associated with the event's payoff. It is hypothesized that on any given trial the individual will choose as if he is attempting to maximize his subjectively expected utility.

The entire development of the extended models will be presented elsewhere.[29] For the present, it is sufficient to say that the derivations follow closely on those of Models I and II presented earlier, the main difference being that subjective probability S_p is utilized in place of the objective probability π that a particular event will occur on any given trial. Thus, where in Model I the predicted asymptote was

$$p = \alpha(\pi - \tfrac{1}{2}) + \tfrac{1}{2} \tag{1}$$

it can be shown that for the extended model the prediction equation becomes

$$p' = \alpha(S_p - \tfrac{1}{2}) + \tfrac{1}{2} \tag{1$_a$}$$

Similarly, the prediction yielded by Model II

$$p = \frac{(1 - \beta) + \pi(\alpha + \beta)}{2} \tag{2}$$

now becomes

$$p' = \frac{(1 - \beta) + S_p(\alpha + \beta)}{2} \tag{2$_a$}$$

For an empirical test of this model, it is necessary to have a theory and a

method for the measurement of subjective probability S_p. We have devised one approach in an experimental test of the prediction $\mathbf{1}_a$ above.

It may be seen from $\mathbf{1}_a$ that when $\alpha = 1.00$, $S_p = p'$. Since there is considerable empirical evidence that α is very close to unity in the No Payoff condition, the behavior of subjects under this condition may be taken as a first estimate of S_p. It may then be assumed that subjective probability is learned in a way that is described by an Estesian exponential function,[18] with $\bar{p}_1$ taken to be 0.50.

In other words, the Estesian learning curve is here reinterpreted as being a representation of the subjective probability function, and the curve is used to obtain S_p for prediction equation ($\mathbf{1}_a$).

Is there any justification for calling an Estesian learning curve a "subjective probability curve"? In general terms, this redefinition transforms the Estes model, an associative model that attaches stimulus "elements" to responses, to a new associative model that attaches stimulus elements to beliefs. In brief, then, the approach may be interpreted to assert: (1) that simulus elements are connected with *beliefs* that an event E_k will or will not occur, (2) that an increase in degree of belief (subjective probability) that E_k will occur results when the belief is confirmed, and (3) that the relation between degree of belief and choice behavior is not a simple one, since choice behavior also reflects what we may call a strategy to maximize subjectively expected utility.

Experimental Test of the Learning Model

In an initial test of the learning model[30] 40 subjects were randomized into two groups of 20 each. Both groups were run in the two-light setup for 200 trials. For these sessions, $\pi = 0.70$, and thus the two lights occurred in the ratio of 70:30, randomized into blocks of 20 trials.

One of the groups of subjects was run under the No Payoff condition. Their responses were used to generate an inferred subjective probability curve.

The other group of 20 subjects was run under conditions of Risk. The average course of learning in this group was predicted from the inferred subjective probability curve obtained from the first group's data and from the learning model.

S_p, taken from each block of the inferred subjective probability curve, was used to derive a predicted learning curve of the group under Risk, by application of

$$p_n' = \alpha(S_{p_n} - \tfrac{1}{2}) + \tfrac{1}{2} \qquad (\mathbf{1}_a)$$

where the value of α was taken from previous research with other subjects under these reinforcement conditions,[27] and was $\hat{\alpha} = 1.718$. FIGURE 1 shows the predicted values and those observed in this study, and reveals that the observed values are in excellent agreement with those predicted from the learning model.

It is interesting to note that because the model assumes that subjective probability is learned independently of the condition of reinforcement, it is possible to obtain data to test the model without having to run a group under No Payoff conditions. That is, a group could be run under any condition of reinforcement for which $\hat{\alpha}$ is known, and then the "no payoff subjective prob-

ability curve" could be derived by

$$S_{p_n} = \frac{p_n' - \frac{1}{2}}{\alpha} + \frac{1}{2}$$

These values could then be used to derive the predicted learning curve for subjects under any other reinforcement conditions.

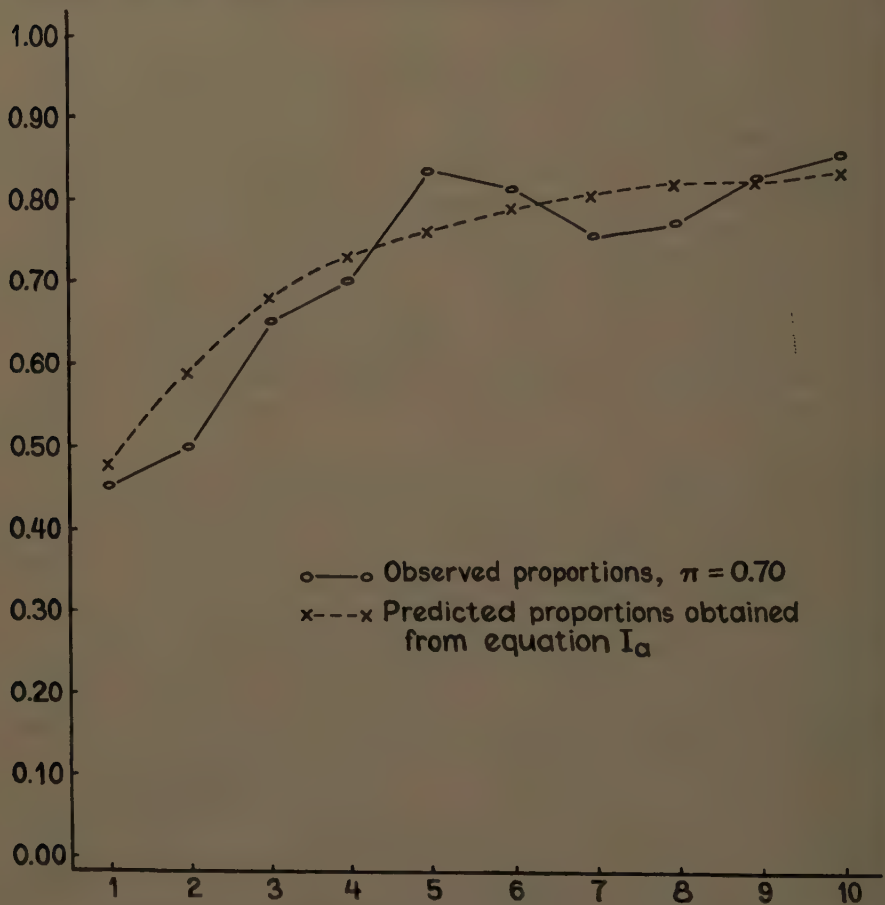

FIGURE 1. The risk condition: observed proportions and predictions.

Summary

From the general hypothesis that humans may be expected to behave as is they were attempting to maximize subjectively expected utility, two modelf have been developed for the asymptotic choice behavior of individuals in a binary choice situation, and a model has been developed to describe the course of learning of strategies in this situation. In these models, the frequency with which the subject predicts the occurrence of the more probable event is taken to represent his strategy, and it is hypothesized that this strategy will vary,

dependent on the various utilities inherent in the situation. For this reason, it is expected that the person's strategy will vary as induced reinforcements are varied, and this expectation has been confirmed in experimentation.

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SOME CHARACTERISTICS OF CHOICE BEHAVIOR IN RISKY SITUATIONS*

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It has been observed that, before deciding between alternatives, people often enumerate the possible outcomes of choosing each alternative. They give each outcome a value representing its desirability, and often form some sort of estimate of the probability of occurrence of each outcome. Scientists have long believed that it should be possible to predict the direction of decision on the basis of these estimates of the value and probability of outcomes. Several models for making such predictions have been proposed (for example, Bernoulli, 1954; Edwards, 1955) based on the concept that people choose the alternative with the greatest expectation (the sum over outcomes of the probability times the value of each outcome). These models are weak because they can be made to predict such a wide range of behavior that it is difficult to design a crucial experiment. This weakness has led us to seek new directions for model building.

Two concepts upon which such a revision might be based are suggested in the current literature, both concerning elements of *risk* in alternatives: (1) "probability preferences," the concept that people are more attracted to certain probabilities of winning or losing than to others (Edwards, 1953); and (2) "variance preferences," the concept that people prefer certain dispersions of values of possible outcomes (Fisher, 1906; Allais, 1953). The present experiment was designed to find out whether probability and variance preferences exist and to discover any interactions that may occur between them. In addition, a model of variance preferences was tested that made more specific predictions than the expected utility interpretation (Bernoulli, 1954), a version of the expected value model.

An earlier report on this study has already been published (Coombs and Pruitt, 1960). In the present paper, the model for preferential choice behavior will be reviewed and the earlier report summarized. Then some additional analyses will be presented concerning inconsistency and transitivity of preference, group scales, and sex differences in probability and variance preferences.

A Model for Variance Preferences

The model is based upon a theory of preferential choice and the unfolding technique discussed in detail elsewhere (Coombs, 1952). Consider a set of alternatives, each having two outcomes at probabilities p and q respectively and all alternatives having the same expected value. The amount of variance of each alternative may be considered a point on a line or continuum of vari-

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ance.* Assuming that an individual has an ideal variance, corresponding to a point on this same continuum, and that he prefers, of any two variances, the one nearer his ideal variance, then his preference ordering of the alternatives is the rank order of the alternatives obtained by folding this scale at his ideal point: the alternatives being ranked in order of increasing absolute distance from his ideal point. It will be convenient to speak of the variance scale generically as a joint scale or J scale (both stimulus points and the ideal points of individuals are located on it); and the preference ordering of an individual, which is a folded J scale, will be called an I scale.

Consider the midpoints between stimuli on the J scale. The order of the midpoints from left to right on the J scale is determined in part by the relative distances between the stimuli. For example, if the distance between the first two stimuli, A and B, is greater than the distance between the next two stimuli, C and D, that is, if $\overline{AB} > \overline{CD}$, then the midpoint of stimuli A and D precedes the midpoint of stimuli B and C. The J scale is divided into segments that are bounded by the midpoints. To each such segment there corresponds a unique ordering of the absolute distances of the stimuli from any point in the segment. This ordering is, by hypothesis, the preference ordering of an individual whose ideal point lies in that segment.

It is evident that while the stimuli may be in the same rank order on the J scale for different individuals, the relative distances between the stimuli may not be the same if the measures of the stimuli are subjective rather than objective. A consequence of this is that the order of the midpoints may be different for different individuals, and the *sets* of preference orderings corresponding to the different orders of midpoints will not be completely identical. It is convenient, therefore, to make the following distinction: if the order of the midpoints appears to be the same for all individuals, the J scale is called a "quantitative J scale" (that is, at least a common ordered-metric scale), whereas if the midpoint order is not the same but only the rank order of the stimuli, the J scale is called a "qualitative J scale" (that is, at most, a common ordinal scale).

In summary, a preference ordering of an individual, called an I scale, is regarded as a folded J scale. If a set of preference orderings, obtained from a number of individuals, may be unfolded to construct a J scale with no contradictory order relations on intervals between stimuli, the J scale is called a quantitative J scale. If the set of I scales may be unfolded to construct a J scale on which only the order of the stimuli is common, the J scale is called a qualitative J scale.

Method

The stimuli were two-outcome bets, such as the following: 1/3 chance to win 50 cents, and 2/3 chance to lose 25 cents. All bets had an expected value of zero. The bets were organized into five sets. Sets I, II, and III each contained 6 bets that differed in variance but had the same probability of winning. In set I, P = 1/3; in set II, P = 1/2; and in set III, P = 2/3. The same six

* In constructing the stimuli of this experiment, variance was defined as $V = pq(a - b)^2$. However, any measure of dispersion is admissible under the model as a definition of variance.

variance levels were used in all three sets: $0.12, 0.30, 1.00, 5.00, 25.00, and 100.00. Sets IV and V each contained 5 bets that differed in probability of winning but had the same variance. The variance of set IV was $1.00; of set V, $25.00. The five differing probabilities were 1/6, 1/3, 1/2, 2/3 and 5/6. Sets I, II, and III were used to study variance preferences under constant probability treatment; sets IV and V were used to study probability preferences under constant variance treatment. Within each set the bets were exhaustively paired, and each pair was presented 8 times.

The bets were given to the subjects in dittoed booklets constructed so that one replication on all pairs was completed before the next was started. The *S*s were told to choose, in each pair, the bet they "would prefer to play" and to indicate this decision by making a check in a box beside the bets. Ninety-nine students at the University of Michigan served as subjects.

Summary of Findings Previously Reported

Consistency and Transitivity of Preference

An information measure was used to eliminate the few *S*s in each set who were essentially random in their choices. This measure provided a convenient index number to characterize an individual's degree of inconsistency of choice in each set of stimuli. Comparison of these indices across sets of stimuli revealed that individual differences in inconsistency are statistically significant and moderate in magnitude.

Weak stochastic transitivity was tested and found to be satisfied for most individuals. When this condition is satisfied, the bets can be simply ordered from left to right for each individual so that each bet is preferred at least one half the time to any bet to the right of it. This is taken to be his preference ordering or *I* scale.

Unfolding Analysis of Preferences

Variance preferences. The preference orderings (*I* scales) for each set of stimuli were unfolded, and a best quantitative *J* scale was constructed. In all three sets (I, II, and III), the vast majority of preference orderings fit the same quantitative *J* scale, and most of those that did not fit deviated in minor ways, apparently as a result of inconsistency. Thus the results gave strong support to the proposed model of variance preferences. However, it was not possible to answer with assurance the question whether a common quantitative or only a common qualitative *J* scale of variance underlies choice. Although the findings seem to support the former conclusion, this might have resulted from the choice of stimuli.

Probability preferences. Again the preference orderings were unfolded for each set. However, in this case, only about 60 per cent of the cases in each set fitted the same quantitative *J* scale, a scale of increasing probability of winning. In most of the preference orderings that did not fit, the trouble seemed to lie with bets at 1/2 chance of winning, which were too much preferred. When these bets were eliminated from the analysis, a much larger proportion of the cases fitted the same quantitative *J* scale of increasing probability of winning.

Stability and Interaction of Preference Patterns

For almost every individual, the probability preference patterns at the two levels of variance (sets IV and V) were remarkably similar, suggesting that probability preferences are stable in the face of changing variance. On the other hand, an individual's variance preferences were often quite different at different probability treatments (sets I, II, and III). These differences were largely a function of probability preferences: at probability levels that people preferred (as shown by their behavior in sets IV and V) they preferred more variance.

TABLE 1
DISTRIBUTION OF $\overline{H}$ WITHIN STIMULUS SETS

$\overline{H}$†	Stimulus sets*				
	Variance preferences			Probability preferences	
	I	II	III	IV	V
0–0.1000	15	37	21	6	7
0.1001–0.2000	14	15	14	14	13
0.2001–0.3000	14	16	9	18	7
0.3001–0.4000	14	12	13	7	18
0.4001–0.5000	13	9	10	12	11
0.5001–0.6000	10	3	10	20	13
0.6001–0.7000	12	3	10	6	15
0.7001–0.8000	1	1	5	8	10
0.8001–0.9000	4	1	3	5	4
0.9001–1.000	2	2	4	3	1
$\overline{\overline{H}}$‡	0.368	0.229	0.367	0.434	0.443

* The entries in all but the bottom row refer to number of *S*s.
† Higher $\overline{H}$ indicates greater inconsistency.
‡ $\overline{\overline{H}}$ is a mean $\overline{H}$.

ADDITIONAL ANALYSES

Inconsistency of Preference

The obtained distribution of the information measure of inconsistency, $\overline{H}$, is given in TABLE 1. The average degree of inconsistency for each of the stimulus sets is contained in the bottom row of the table, indicating that probability preferences were most inconsistent, variance preferences at P's of 1/3 and 2/3 next most inconsistent, and variance preferences at 1/2 most consistent.

In a previous study of inconsistency of preferences (Coombs, 1958) it was shown that laterality of stimuli on the *J* scale relative to an individual's ideal affected the order of magnitude of his inconsistency. For individuals toward the extreme of a *J* scale the pairs of stimuli are predominantly unilateral, and for individuals with intermediate ideals, there are more bilateral pairs. Variability in the location of an individual's ideal inflates the inconsistency of bilateral judgments but has no effect on unilateral judgments. Since all individuals are assumed to have variable ideals, one would expect the inconsistency measure of an individual's preference to be higher the nearer he is to the middle

of the J scale. These conclusions are borne out by the results presented in TABLE 2. Stimulus sets I, II, and III have 6 stimuli each, designated A to F in order of increasing variance, in this table. Sets IV and V have only 5 stimuli, and the symbols A to E correspond to the probabilities 1/6 to 5/6 respectively. It is evident that, for each set of stimuli, inconsistency is greatest for individuals centrally located on the J scale and diminishes toward either end.

TABLE 2
RELATION OF INCONSISTENCY TO IDEAL

	Stimulus sets														
	I			II			III			IV			V		
	$\bar{\bar{H}}$*	Md $\bar{H}$†	N‡	$\bar{\bar{H}}$	Md $\bar{H}$	N	$\bar{\bar{H}}$	Md $\bar{H}$	N	$\bar{\bar{H}}$	Md $\bar{H}$	N	$\bar{\bar{H}}$	Md $\bar{H}$	N
Most preferred stimulus															
A	0.2157	0.1630	34	0.1802	0.1450	18	0.2582	0.1948	16	0.4070	0.3442	33	0.4566	0.4895	31
B	0.4045	0.3959	4	0.2167	0.2189	4	0.3500	0.3841	13	0.6252	0.5807	3	0.6884	0.7401	4
C	0.4520	0.4979	16	0.2846	0.2657	11	0.5689	0.5743	8	0.5231	0.5173	21	0.5366	0.5295	15
D	0.4884	0.4824	12	0.4138	0.4273	12	0.5188	0.5452	14	0.4288	0.4341	9	0.4512	0.4053	6
E	0.4586	0.4471	9	0.2547	0.2627	15	0.3626	0.3472	5	0.2978	0.3253	29	0.3341	0.3397	38
F	0.2112	0.2169	17	0.1095	0.0362	36	0.2359	0.1812	37						

* $\bar{\bar{H}}$ is the mean $\bar{H}$.
† $\bar{H}$ indicates inconsistency.
‡ N indicates number of *S*s.

Strong Stochastic Transitivity

Whether inconsistent preferential choices satisfy strong stochastic transitivity is of some interest. Strong stochastic transitivity (s.s.t.) is defined as follows for any triple (a, b, c):

$$P(a,b) \geqslant 0.5 \text{ and } P(b,c) \geqslant 0.5 <=> P(a,c) \geqslant \max\{P(a,b), P(b,c)\}$$

It is clear that data satisfying this condition would also satisfy the condition for weak stochastic transitivity. The distinction is made clear in this example. Suppose that $P(a,b) = 0.60$ and $P(b,c) = 0.70$. If $P(a,c)$ is at least 0.50 then weak stochastic transitivity is satisfied by the ordering *abc*. Strong stochastic transitivity requires that $P(a,c)$ be at least 0.70. It is obvious that a monotone transformation of inconsistency into psychological distance, as in the Thurstone (1927) and Luce (1959) models, requires strong stochastic transitivity.

While no simple statistical test exists for testing whether the data of this experiment satisfy strong stochastic transitivity, a simple tabulation of the degree to which they do is of interest. This tabulation is presented in TABLE 3. Strong stochastic transitivity is satisfied by only a small number of those subjects whose preference orderings satisfy weak stochastic transitivity (column *b*). It must be kept in mind that each pairwise probability is based on only eight observations and it is to be expected that even if strong stochastic transi-

tivity held in principle, a certain number of *S*s would fail to satisfy it. Unfortunately no statistical test may readily be made of whether the observed number is a significant departure: one only forms the impression that this may be so.

Further evidence is contained in the number of triples that fail to satisfy strong stochastic transitivity. In stimulus sets I, II, and III, there are 20 triples imbedded in the weak stochastic ordering of the 6 stimuli in each set. In sets IV and V, with 5 stimuli, there are 10 triples in each. The failure of an individual's ordering to satisfy strong stochastic transitivity is due to the failure of one or more of these triples. Column *c* of TABLE 3 contains the total number of triples, in each stimulus set, multiplied by the number of *S*s. Column *d* gives the number of such triples that fail to satisfy strong stochastic transitivity and column *e* indicates the proportion. It is apparent that, on the average, about 25 per cent of all triples fail to satisfy strong stochastic

TABLE 3
NUMBER OF *S*S AND TRIPLES SATISFYING STRONG STOCHASTIC TRANSITIVITY

Stimulus set	*a*	*b*	*c*	*d*	*e*	*f*
I	86	9	1720	442	0.26	5.74
II	91	31	1820	308	0.17	5.13
III	85	14	1700	453	0.27	6.38
IV	89	18	890	219	0.25	3.08
V	88	13	880	243	0.28	3.24

(*a*) Total number of *S*s satisfying weak stochastic transitivity.
(*b*) Total number of *S*s satisfying strong stochastic transitivity.
(*c*) Total number of triples in the weak stochastic orderings.
(*d*) Total number of triples failing to satisfy strong stochastic transitivity.
(*e*) Proportion of triples failing to satisfy strong stochastic transitivity.
(*f*) Mean number of triples failing to satisfy strong stochastic transitivity for those *S*s who depart from strong stochastic transitivity.

transitivity. For those *S*s who fail to satisfy strong stochastic transitivity, the average number of triples in their rank order that violate this condition is given in the last column of the table. This indicates that failure to satisfy strong stochastic transitivity is not a consequence of just one or two triples but is perhaps a significant departure.

Considerations of the laterality of stimuli on the *J* scale relative to an individual's ideal (Coombs, 1958) also suggest that strong stochastic transitivity would be less often satisfied by people whose ideals are nearer the middle of the distribution. Rather than present all the results in detail a summary is contained in TABLE 4. All variance preference *I* scales are divided into three groups, those beginning with one of the end stimuli (A or F) and those beginning with an intermediate stimulus. The number of such preference scales and the number of triples in their preference scales that satisfy strong stochastic transitivity are presented in TABLE 4. As is evident, the expectation is borne out. The results on probability preferences show the same trend, but not as strongly, probably due to the fact that there are fewer stimuli (and hence less effect of laterality), and because of the strong preference for P = 1/2.

Group Preferences

Preference scales based on the pooled data for all individuals in a stimulus set are of some interest. A group preference scale may be constructed by majority choice on each pair if weak stochastic transitivity holds for the group, with each individual getting one vote on each pair. The result is a rank order that is a maximum likelihood preference scale for the entire group. The results are presented in TABLE 5.

Each of these group scales may be seen to correspond to an *I* scale and the corresponding *I* scale number may be obtained by counting the number of

TABLE 4
NUMBER OF SUBJECTS AND TRIPLES SATISFYING STRONG STOCHASTIC TRANSITIVITY RELATED TO IDEAL

	a	*b*	*c*	*d*	*e*	*f*
A	64	14	0.219	1280	1035	0.809
B to E	106	7	0.066	2120	1513	0.714
F	83	32	0.386	1660	1361	0.820

(*a*) Number of *I* scales.
(*b*) Number of *S*s satisfying strong stochastic transitivity.
(*c*) Proportion of *S*s satisfying strong stochastic transitivity.
(*d*) Total number of triples.
(*e*) Number of triples satisfying strong stochastic transitivity.
(*f*) Proportion of triples satisfying strong stochastic transitivity.

TABLE 5
GROUP PREFERENCE SCALES

	Stochastically-dominant group preference scale	Corresponding *I* scale number
Probability treatment		
P = 1/3	CBADEF	4
P = 1/2	EDCBAF	11
P = 2/3	DECBAF	10
Variance treatment		
Low	32415	5
High	34521	8

midpoints crossed to obtain the group scale and adding *one*. These numbers are given in the last column of the table. It is evident that, as a group, maximum variance is preferred at 50–50 odds and more variance is preferred at favorable than at unfavorable odds. At higher variance the group tends to prefer odds more in their favor, although the prior analysis of variance (Coombs and Pruitt, 1960) suggests that this is not a significant difference. In all cases the group scales are folded *J* scales and are different under different treatments.

Another way of characterizing the preferences of a group is through the distribution of the ideals (first choices) of its members. Such distributions are shown in the histograms of FIGURES 1 and 2. In all cases it will be seen that the distributions are U-shaped, with roughly one third of the subjects favoring each of the two extreme stimuli and one third favoring the intermediate stimuli.

A method frequently used for obtaining a group scale is to determine the total number of times each stimulus has been preferred in its pairwise presentation with all others. This vote-count method for preferential choice has been shown to be not independent of other alternatives (Coombs, 1958) when preferences are generated from a joint scale, and it has not been used here.

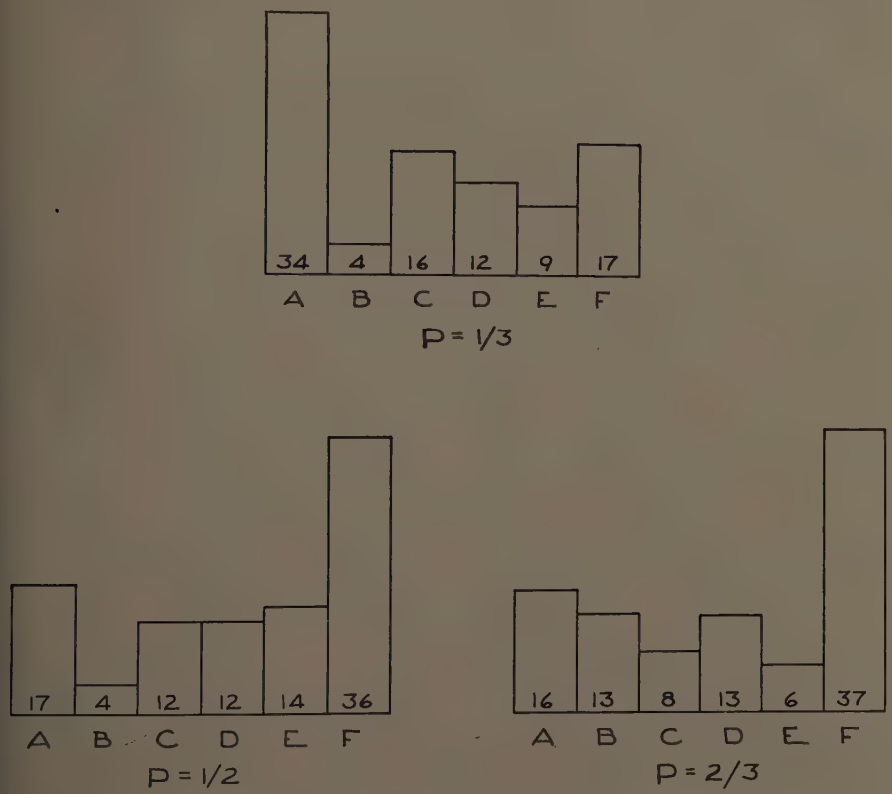

FIGURE 1. Frequency of first choices in variance preferences.

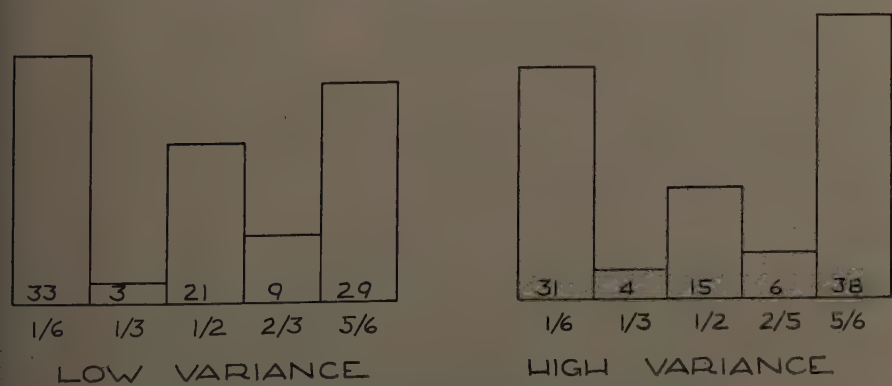

FIGURE 2. Frequency of first choices in probability preferences.

Sex Differences

Inconsistency. In TABLE 6 is shown the mean $\bar{H}$ for men and women separately on variance and probability preferences, and TABLE 7 contains an analysis of variance. It is clear that the men are more consistent than the women in their preference behavior in the case of both variance and probability preferences. There is no significant interaction between sex and treatments in the case of either variance or probability preferences.

Transitivity. These differences are not worth reporting in detail. There are slightly more intransitive *I* scales, and *I* scales with ties, for women than for men, but these may be attributed to the greater inconsistency of the women.

Preference. A distribution of the first choices of men and women in their variance preferences reveals a significant tendency for women to fold the *J* scale of variance, whereas men are more evenly distributed. This is borne out in the 3 fourfold tables presented in TABLE 8.

TABLE 6
MEAN $\bar{H}$ FOR VARIANCE AND PROBABILITY PREFERENCES OF MEN AND WOMEN

Sex	Stimulus sets				
	I	II	III	IV	V
Male	0.3203	0.1902	0.3114	0.3834	0.3803
Female	0.4025	0.2570	0.4083	0.4708	0.4893

TABLE 7
TEST OF SEX DIFFERENCES IN INCONSISTENCY OF VARIANCE AND PROBABILITY PREFERENCES

Source	Variance preferences			Probability preferences		
	df	*MS*	*F*	*df*	*MS*	*F*
Sexes	1	0.4872	4.091*	1	0.4561	4.925*
Error	97	0.1191		97	0.0926	

Note: interaction of sexes and probability treatments not significant by inspection.
* Significant beyond the 0.05 level.

The probability preferences also present a trend in sex differences, and the relevant data are contained in TABLE 9. Women tend to prefer "fair" bets and positively-skewed bets more than men, whereas the men seem to prefer the negatively-skewed bets more: these are the "almost sure things" with small probability of a large loss.

CONCLUSIONS

Findings on variance preferences give strong support to a model which asserts that, at each probability of winning, the individual has an ideal level of variance and will choose, of any two levels of variance, that which is closer to his ideal. A similar model for probability (skewness) preferences appears to hold, except that 50–50 bets are excessively preferred.

A vast proportion of the *S*s were significantly consistent in their preferences and exhibited weak stochastic transitivity. On the other hand, strong stochastic transitivity was not met in most cases. Inconsistency of preferences and failure to satisfy strong stochastic transitivity were observed to be greater for individuals whose ideals are intermediate on the *J* scale, compared with those whose ideals are extreme. This is to be expected from the effect of laterality on the inconsistency of preferences.

TABLE 8
SEX DIFFERENCES IN VARIANCE PREFERENCES

	Stimulus sets					
	I		II		III	
	Male	Female	Male	Female	Male	Female
Var. prefs.						
Extreme	27	24	28	25	26	28
Folded	13	28	13	30	13	26
Test	$\chi^2 = 4.125$ 1 *df* $P < 0.05$		$\chi^2 = 5.011$ 1 *df* $P < 0.05$		n.s.	

TABLE 9
SEX DIFFERENCES IN PROBABILITY PREFERENCES

	Stimulus sets			
	IV: Low variance		V: High variance	
	Male	Female	Male	Female
1/6 and 1/3	16	20	13	22
1/2	4	17	2	13
2/3 and 5/6	22	16	26	18
Test	$\chi^2 = 6.71$, 2 *df* $P < 0.05$		$\chi^2 = 8.51$, 2 *df* $P < 0.02$	

Group scales based on majority choice were folded *J* scales, and they revealed that for the group as a whole, maximum variance was preferred at odds of one half, and more variance was preferred at favorable than at unfavorable odds; at the higher variance level the group tended to prefer odds in their favor. Distributions of first-choice were U-shaped for all sets of stimuli.

Women differed from men in that they (1) were more inconsistent, (2) were correspondingly less often transitive, (3) preferred intermediate variance more often than men, and (4) preferred even money and long shot bets more than men. It is not clear whether women's greater inconsistency is due to the laterality effect, in that their ideals tend to be intermediate, or whether they may be said to be intrinsically more inconsistent in this kind of behavior.

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Part III. Empirical Research Evidence Regarding Decision Making

STOCHASTIC LEARNING IN RATS WITH HYPOTHALAMIC IMPLANTS

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Introduction

Several investigators have conducted learning experiments with humans and with animals designed to test the validity of various stochastic* models for learning. Estes[1] pioneered in this field. Independently, but at about the same time, very similar theoretical approaches were being developed by Bush and Mosteller,[2] by Miller and McGill,[4] and by Bales *et al.*[5]

The outstanding work of Bush and Mosteller, especially on the important problems of statistical estimation with applications to experimental data of various workers, was reported fully in their book on *Stochastic Models for Learning*.[6] Recent work by Burke, Bush, Estes, Mosteller, and others is reported in a volume entitled *Studies in Mathematical Learning Theory*, edited by Bush and Estes.[7] Especially noteworthy, more recently, is the theoretical approach of Luce,[8] as reported in his book, *Individual Choice Behavior*, and in the work of Suppes[9] and others at Stanford University, Stanford, Calif.

The present paper reports on intermittent work, started in 1951, with one particular stochastic learning model that I have called the "symmetrical model." The characteristics of this mathematical model that are essential for the present paper are summarized in the next section. The symmetrical model has been compared with some other stochastic models in an earlier paper of mine[3] in which a comparison was made particularly with the combining-of-classes model of Bush and Mosteller. Critical aspects of experimental and theoretical procedure and meanings are discussed there also. Suffice it to say that, for present purposes, the symmetrical model seems worth checking experimentally with respect to its range of validity. I have called this range of validity the "scope" of the theory.

Although the work is discussed here in the context of individual learning in rats, my interest in it has been and is primarily with reference to organization theory. A series of earlier papers[10] deal with the use of stochastic models of this kind as models for decision-making groups of interacting individuals. No attempt is made in the present paper to indicate further the use of such stochastic models in organization science, so that it is best read as a straightforward paper on individual learning.

The motivation for using rats, rather than human subjects, is threefold:

(1) Close control, and use of extremes of reward and punishment is possible with rats.

* Estes[1] referred to his model as "statistical" or "probabilistic," and Bush and Mosteller[2] to theirs as "statistical" or "mathematical." I have called[3] the approach "stochastic learning theory," because the mathematical models used are ordinary stochastic processes; "stochastic learning" is now a term quite commonly used.

(2) Implanted electrodes in the brain of the rat can be used to provide stimuli that are either rewarding or punishing, and readings from electrodes can provide useful information regarding the rat's behavior.

(3) Experimental costs and difficulties are less with rats than with human subjects when they are utilized for extensive observation over a considerable period of time.

The experiments reported here were all conducted with rats having chronic implants in the hypothalamic region, so that the rat was rewarded by electrical stimulation at the inner tip of the electrode. The rat made his choices by pressing pedals arranged rather symmetrically within a cage, and stimulation followed immediately upon pressing a pedal if the experimenter's pattern of rewards so required. Details of the apparatus and experimental techniques will be presented in a separate paper; a summary is included in *Appendix A*.

The prime objective of the present experimentation is to determine whether or not a procedure can be devised for measuring the six behavioral parameters of the symmetrical stochastic learning model in a reliable manner for an individual rat. A later objective, granted success with the present objective, is the determination of rough limits on the scope of the theory. For example:

(1) Do the measured behavioral parameters remain reasonably constant for an individual rat with changes in such experimental variables as number of pedals available, strength of electrical stimulation, location of implant, type of reward (such as food versus implant), alteration by surgery or drugs, and age?

(2) Are the measured behavioral parameters adequate to pass predictively from free pace to forced pace conditions, from free pace to latency, and so on?

(3) Do the behavioral parameters vary from rat to rat, and are the parameters related to other psychophysical measures?

Concurrently with the rat experiments, there is parallel experimentation with human subjects utilizing monetary rewards rather than chronic implants; these experiments will be reported elsewhere.

The present paper reports on the analytical techniques used to estimate behavioral parameters, and discusses some preliminary experimental results obtained with two rats. It is concluded by my colleagues and me that the behavioral parameters can be estimated reliably. No attempt has yet been made to determine the range of validity of these estimates, or the scope of the theory, as the experimental conditions are changed.

Symmetrical Model

The symmetrical stochastic learning model is described by the following mathematical relations. We start with a more general stochastic model,

$$p(t+1) = M^{r_t s_t} p(t), \tag{1}$$

where $p(t)$ is the m-dimensional stochastic vector whose r^{th} component is a probability $p_r(t)$, and where M^{rs} is a square stochastic matrix of order m whose elements M^{rs}_{ij} depend only upon r and s. It is supposed that the subject makes a sequence of responses among a fixed finite set of alternatives, and that there is a probability $p_r(t)$, at the end of moment t, that response r will occur before the end of moment $(t+1)$. The notation is such that r_t denotes the response

actually made after moment t, and s_t denotes the outcome (stimulus) following response r_t.

The symmetrical model is the special case of **1**, in which the following conditions are satisfied:*

$$M_{ij}^{rs} = a^s\delta_{ij}\delta_{ri} + c^s\delta_{ij}(1 - \delta_{ri}) + \frac{(1 - a^s)}{m - 1}(1 - \delta_{ri})\delta_{rj} + b^s(1 - \delta_{rj})\delta_{ri} + \mu(1 - b^s - c^s)(1 - \delta_{ij})(1 - \delta_{ri})(1 - \delta_{rj}) \tag{2}$$

for $r = 1, 2, \cdots, m$; $s = 1, 2, \cdots, n$,

where

$$\mu = \begin{cases} 1/(m - 2) & \text{if } m > 2 \\ 0 & \text{if } m = 2, \end{cases}$$

and where

$$\begin{aligned} 0 &\leq a^s \leq 1, \\ 0 &\leq b^s \leq 1, \\ 0 &\leq c^s \leq 1, \\ 0 &\leq b^s + c^s \leq 1. \end{aligned} \tag{3}$$

The quantities a^s, b^s, and c^s are the three behavioral parameters corresponding to outcome s; thus, there are $3n$ behavioral parameters in all, since there are n outcomes possible.

It is easily seen that **1** and **2** are equivalent to the following conditions:

$$\begin{cases} p_{r_t}(t + 1) = A^s p_{r_t}(t) + B^s, \\ p_r(t + 1) = C^s p_{r_t}(t) + D^s p_r(t) + E^s \quad \text{for } r \neq r_t, \end{cases} \tag{4}$$

where

$$\begin{aligned} A^s &\equiv a^s - b^s, \\ B^s &\equiv b^s, \\ C^s &\equiv \left(\frac{1 - a^s}{m - 1} - \frac{1 - b^s - c^s}{m - 2}\right), \\ D^s &\equiv \left(c^s - \frac{1 - b^s - c^s}{m - 2}\right), \\ E^s &\equiv \left(\frac{1 - b^s - c^s}{m - 2}\right). \end{aligned} \tag{5}$$

In other words, the probability of a response r_t being repeated is a constant B^s, plus its previous probability multiplied by a constant A^s; and the probability of a new response r is a constant E^s, plus its previous probability multiplied by a constant D^s, plus the previous probability of the previous response r_t multiplied by a constant C^s.

* δ_{xy} is the familiar Kronecker delta, such that: $\delta_{xy} = 1$ if $x = y$, and $\delta_{xy} = 0$ if $x \neq y$.

It has been shown[3] that the symmetrical learning model satisfies the combining-of-classes condition of Bush and Mosteller if and only if the behavioral parameters satisfy the following additional conditions:

$$C^s = 0, \text{ for } s = 1, 2, \cdots, n. \tag{6}$$

In this special case, the equations **4** reduce to the following:

$$p_{r_t}(t+1) = (a^s - b^s)p_{r_t}(t) + b^s,$$

$$p_r(t+1) = (a^s - b^s)p_r(t) + (1 - a^s)/(m-1), \text{ for } r \neq r_t.$$

However, there are other stochastic learning models of the form **1** that are not symmetrical but do nevertheless satisfy the combining-of-classes condition;[3] these models are not considered in the present paper.

Parameter Estimation

Experimental data always consist of observations on r_t and s_t, for $t = 1, 2, \cdots, N$. The possible values of r_t are $r_t = 1, 2, \cdots, m$; the possible values of s_t are $s_t = 1, 2, \cdots, n$; and N denotes the number of successive responses in one experimental trial. Of course, the trials may be repeated, with N different in each case.

In addition to the behavioral parameters a^s, b^s, and c^s, there are also the m parameters $p(1)$, which we shall refer to as the "state parameters." The state parameters describe the subject at the beginning of any experimental trial. It is equally proper to speak of the "state of the subject at moment t" as the stochastic vector $p(t)$; however, for convenience, we shall use state parameters to indicate the starting state for a particular experimental trial.

Maximum-likelihood estimates would be adequate, for our purposes, but are often difficult to obtain. For data r_t and s_t from a single experimental trial, these maximum-likelihood estimates would be defined by the following relations. First, we define quantities $\hat{p}(t)$ recursively as follows:

$$\hat{p}(t+1) \equiv \hat{M}^{r_t s_t} \hat{p}(t) \quad \text{for} \quad t = 1, 2, \cdots, N, \tag{7}$$

where $\hat{M}_{ij}^{rs}$ and $\hat{p}(1)$ are defined to be the values of the parameters M_{ij}^{rs} and $p_k(1)$ that maximize the "likelihood"

$$L[M_{ij}^{rs}, p_k(1)] \equiv \prod_{t=1}^{N} \hat{p}_{s_t}(t), \tag{8}$$

where the estimated parameters $\hat{M}_{ij}^{rs}$ and $\hat{p}(1)$ are required also to satisfy the restrictions of the symmetrical learning model. A numerical example is given in the next section to illustrate this estimation procedure.

If several trials are available for analysis, perhaps with different starting states and lengths, then the likelihood estimates would be the values of M_{ij}^{rs} and $p_k^h(1)$ that maximize the likelihood

$$L[M_{ij}^{rs}, p_k^h(1)] = \prod_{l=1}^{q} \prod_{t=1}^{N_l} \hat{p}_{r_t l}^{l}(t), \quad h = 1, 2, \cdots, q; \quad k = 1, 2, \cdots, m; \tag{9}$$

where q is the number of separate trials and N_l is the length of trial l. In this case, of course, the quantities $\hat{p}_{r_t l}^{l}(t)$ are defined as follows, analogous to the definition in **7**:

$$\hat{p}^l(t+1) \equiv \hat{M}^{r_t l s_t l} \hat{p}^l(t) \quad \text{for} \quad (t = 1, 2, \cdots, N_l; \quad l = 1, 2, \cdots, q). \tag{10}$$

In many instances, the experimenter will determine the sequences of outcomes of s_t^l in advance of the trials. If these sequences are chosen suitably, the calculations required to obtain usable estimates of the parameters can be reduced greatly. Furthermore, the state parameters may sometimes be determined by the nature of the experiment so that they need not be estimated from the experimental data. We now consider the estimation problem for one design of experiment that takes advantage of such simplification. This method is illustrated by a numerical example in the next section, but is not used extensively in the present paper.

If the experiment is designed to require repeated trials, each with the same sequence of outcomes and each with the starting state parameters all equal, then the parameters may be estimated rather easily. Specifically, for convenience, we start by considering the following design:

$$\begin{aligned} &N_1 = N_2 = \cdots = N_q = 3, \\ &\hat{p}_1^l(1) = \hat{p}_2^l(1) = \cdots = p_m^l = (1/m), \quad \text{for} \quad (l = 1, 2, \cdots, q), \\ &s_t^l = 1, \quad \text{for} \quad (l = 1, 2, \cdots, q; \qquad t = 1, 2, 3). \end{aligned} \tag{11}$$

Furthermore, for convenience, we start by considering the case in which

$$m = 3, \qquad n = 2. \tag{12}$$

Thus, the subject chooses one from among three possible choices for a sequence of three responses in each of q trials, and the outcome is always the same from among two possible outcomes (for example, outcome No. 1). As a further matter of notation, since we shall presently be concerned only with the parameters a^1, b^1, c^1, from the matrices M^{r1}, we shall temporarily drop the superscripts corresponding to s and have simply:

$$\begin{aligned} M^1 &\equiv \begin{Vmatrix} a & b & b \\ \frac{1-a}{2} & c & 1-b-c \\ \frac{1-a}{2} & 1-b-c & c \end{Vmatrix} \\ M^2 &\equiv \begin{Vmatrix} c & \frac{1-a}{2} & 1-b-c \\ b & a & b \\ 1-b-c & \frac{1-a}{2} & c \end{Vmatrix} \\ M^3 &\equiv \begin{Vmatrix} c & 1-b-c & \frac{1-a}{2} \\ 1-b-c & c & \frac{1-a}{2} \\ b & b & a \end{Vmatrix}. \end{aligned} \tag{13}$$

There are actually $3^3 = 27$ possible sequences of responses that the subject can make in one trial with three responses, each from among three possible alternatives represented by all the possible arrangements of the integers 1, 2, and 3. However, because of the symmetry of the stochastic model in use, it is easily seen that there are only five types of sequences that require separate consideration in the analysis; this is so because the first response in a trial may be arbitrarily labeled 1, the first subsequent response that is different (if any) may be arbitrarily labeled 2, and the remaining distinct response (if any) may be labeled 3. Hence, the five distinctive sequences of responses are:

111 (includes 222 and 333),
112 (includes 113, 221, 223, 331, and 332),
121 (includes 131, 212, 232, 313, and 323),
122 (includes 133, 211, 233, 311, and 322),
123 (includes 132, 213, 231, 312, and 321).

The experimental data may therefore be completely described by five frequencies, as follows:

F_{111} = the number of times the first response is twice repeated,
F_{112} = the number of times the first response is once repeated,
F_{121} = the number of times the second response differs from the first response when the third response is the same as the first response, and
F_{121} and F_{123} are defined similarly.

Of course,

$$F_{111} + F_{112} + F_{121} + F_{122} + F_{123} = q.$$

With this notation, by substitution in **9**, we can obtain the likelihood function for this particular design of experiment when it yields observations F_{ijk}. We shall now derive such an expression for $L[a, b, c]$, after some preliminary calculations. Consider first the case F_{111}. The theoretical probability of p_{111} of observing response patterns 111, 222, or 333, is as follows:

$$p_{111} = P_{111} + P_{222} + P_{333} = 3P_{111},$$

where P_{ijk} denotes the theoretical probability of observing the single sequence (i, j, k). Also, since $p_k^l(1) = (\frac{1}{3})$, for $(l = 1, 2, \cdots, q; k = 1, 2, 3)$, we shall temporarily omit the superscript on $p^l(1)$.

We adopt the customary notation that a prime on a matrix denotes the transposed matrix. Since $p(t)$ and $p(1)$ are matrices with one column, or equivalently column vectors, then $p'(t)$ and $p'(1)$ denote their transposition; thus, $p'(1) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in our present discussion. It will also be convenient to let e_h denote the unit column vector with m rows, having unity in its hth row and zeros elsewhere; and to let J denote the column vector with m rows and all entries unity. Thus, also, $p(1) = J/3$ and $p'(1) = J'(3)$.

The theoretical probability of response i at moment t is therefore $e_i'p(t)$. Consequently,

$$P_{ijk} = [e_i'p(1)][e_j'M^ip(1)][e_k'M^jM^ip(1)] = [p_i(1)][p_j(2)][p_k(3)]. \qquad \textbf{(14)}$$

We now proceed to calculate P_{111}, P_{112}, P_{121}, P_{122}, P_{123}, that correspond to the observational data F_{ijk}. It follows easily that:

$$M^1p(1) = \tfrac{1}{3}\begin{Vmatrix} a + 2b \\ (3 - a - 2b)/2 \\ (3 - a - 2b)/2 \end{Vmatrix} = \begin{Vmatrix} p_1(2) \\ p_2(2) \\ p_3(2) \end{Vmatrix} \equiv \begin{Vmatrix} w \\ (1 - w)/2 \\ (1 - w)/2 \end{Vmatrix}, \tag{15}$$

where $p_1(2) \equiv w$ denotes the theoretical probability of response 1 at moment 2. Similarly:

$$M^1M^1p(1) = \tfrac{1}{6}\begin{Vmatrix} 2a^2 - 4b^2 + 2ab + 6b \\ -a^2 + 2b^2 - ab - 3b + 3 \\ -a^2 + 2b^2 - ab - 3b + 3 \end{Vmatrix}$$

$$= \begin{Vmatrix} 3w^2 - 3bw + b \\ (-3w^2 + 3bw - b + 1)/2 \\ (-3w^2 + 3bw - b + 1)/2 \end{Vmatrix} = \begin{Vmatrix} p_1(3) \\ p_2(3) \\ p_3(3) \end{Vmatrix}, \tag{16}$$

$$M^2M^1p(1) = \tfrac{1}{12}\begin{Vmatrix} a^2 + 4b^2 + 4ab + 6ac + 12bc - 6a - 12b - 6c + 9 \\ -2a^2 + 4b^2 - 2ab + 6a + 6b \\ a^2 - 8b^2 - 2ab - 6ac - 12bc + 6b + 6c + 3 \end{Vmatrix}$$

$$= \tfrac{1}{4}\begin{Vmatrix} 1 + 3w^2 + 2(1 - 3w)(1 - c) \\ 2 - 6w^2 - 2(1 - 3w)(1 + b) \\ 1 + 3w^2 + 2(1 - 3w)(b + c) \end{Vmatrix} = \begin{Vmatrix} p_1(3) \\ p_2(3) \\ p_3(3) \end{Vmatrix}. \tag{17}$$

A straightforward calculation shows that:

$$p_{111} = 3P_{111} = 3[\tfrac{1}{3}][w][3w^2 - 3bw + b] = 3w^3 - 3bw^2 + bw. \tag{18}$$

Similarly:

$$\begin{aligned} p_{111} &= w[3w^2 + (1 - 3w)b], \\ p_{112} &= w[(1 - 3w^2) - (1 - 3w)b], \\ p_{121} &= \left(\frac{1 - w}{2}\right)\left[\frac{1 + 3w^2}{2} + (1 - 3w)(1 - c)\right], \\ p_{122} &= \left(\frac{1 - w}{2}\right)[1 - 3w^2 - (1 - 3w)(1 + b)], \\ p_{123} &= \left(\frac{1 - w}{2}\right)\left[\frac{1 + 3w^2}{2} + (1 - 3w)(b + c)\right]. \end{aligned} \tag{19}$$

Here, for example, we have used the fact that $p_{121} = 6P_{121}$ since the six equally likely actual sequences (121, 131, 212, 232, 313, 323) are all included under p_{121}.

The logarithm of the likelihood function **9** is, in this special case:

$$ln(L[a, b, c]) = q\left[f_{111}ln\left(\frac{p_{111}}{3}\right) + f_{112}ln\left(\frac{p_{112}}{6}\right) + f_{121}ln\left(\frac{p_{121}}{6}\right) + f_{122}ln\left(\frac{p_{122}}{6}\right) + f_{123}ln\left(\frac{p_{123}}{6}\right)\right]$$

$$= q[f_{111}ln\, p_{111} + f_{112}ln\, p_{112} + f_{121}ln\, p_{121} + f_{122}p_{122} + f_{123}p_{123} - ln6 + f_{111}ln2],$$

where $f_{ijk} \equiv (F_{ijk})/q$ are the relative frequencies. The parameter estimates that we seek, therefore, are the quantities $\hat{a}$, $\hat{b}$, $\hat{c}$ that maximize $ln\, L$ but are subject to the restrictions of **3**.

Useful parameter estimates might be found by solving the following system of equations and inequalities, if such a solution exists:

$$\begin{aligned} &p_{111} = f_{111}, \quad p_{112} = f_{112}, \quad p_{121} = f_{121}, \quad p_{122} = f_{122}, \quad p_{123} = f_{123}, \\ &0 \le a \le 1, \quad 0 \le b \le 1, \quad 0 \le c \le 1, \quad 0 \le b + c \le 1. \end{aligned} \tag{20}$$

Of course, the system of relations **20** should have a solution if the type of stochastic model used is the one yielding the observations and if the number of observations is great enough. When the system **20** does not have a solution, because of the inequalities, it can be used to obtain initial values to be used in an iterative procedure for determining actual likelihood estimates.

It is quite easy to find the explicit solution of **20**, if it exists, as follows:

$$\begin{aligned} \hat{w} &= f_{111} + f_{112}, \\ \hat{c} &= 1 + \frac{2}{(1 - \hat{w})(1 - 3\hat{w})}\left[\frac{(1 + 3\hat{w}^2)(1 - \hat{w})}{4} - f_{121}\right], \\ \hat{b} &= -1 + \frac{2}{(1 - \hat{w})(1 - 3\hat{w})}\left[\frac{(1 - 3\hat{w}^2)(1 - \hat{w})}{2} - f_{122}\right], \\ \hat{a} &= 3\hat{w} - 2\hat{b}. \end{aligned} \tag{21}$$

After $\hat{a}$, $\hat{b}$, and $\hat{c}$ are computed according to **21**, it is only necessary to check that the values found satisfy the other requirements of **20** in order to ensure that they constitute the desired solution; this means checking the inequalities and the equations for f_{111} and f_{123}.

Exactly the same formulas apply in any case when a, b, c are abbreviations for a^s, b^s, c^s, whatever the value of s, provided only that s remains constant throughout the set of trials analyzed. Accordingly, independent estimates of a^s, b^s, and c^s may be obtained by selecting from among all the trials just those for which s has the desired value for the first two responses.

Formulas exactly analogous to **21** can also be found, in a straight-forward manner, when there are more than three responses in the replicated trials.

For example, in the case of four responses per trial, the observational data would be the following set of relative frequencies:

$$f_{1111}, \quad f_{1112}, \quad f_{1121}, \quad f_{1122}, \quad f_{1123}, \quad f_{1211}, \quad f_{1212}, \quad f_{1213}, \quad f_{1221}, \quad f_{1222},$$
$$f_{1223}, \quad f_{1231}, \quad f_{1232}, \quad f_{1233}.$$

The 14 theoretical probabilities p_{ijkl}, that correspond to these 14 observed relative frequencies, are each a function of the parameters a, b, c. Some subset of the 14 equations $p_{ijkl} = f_{ijkl}$ could be used to determine estimates of a, b, and c; again, it would be necessary to check that these estimates satisfy the requirements of **20**. Alternatively, the likelihood function could be used to obtain estimates in this case. It is already quite clear that the algebraic and computational complexities mount very rapidly as the number of responses is increased beyond 3 or 4 per trial.

Formulas analogous to **21** can also be found, in a straightforward but tedious manner, if the value of s_t is not constant over the set of responses but the sequential pattern of s_t is the same for each trial. For example, if the values of s_t are (1, 2, 1), then the case of four responses per trial would yield 14 equations between p_{ijkl} and f_{ijkl} from which the six parameters could be estimated. On the other hand, if the values of s_t were (1, 1, 2), then the first three responses would yield estimates of the parameters corresponding to $s = 1$, and the four-response data could be used to estimate the remaining three parameters corresponding to $s = 2$. As always, maximum-likelihood estimates represent a superior alternative procedure when the calculations are feasible.

A fundamentally different method of estimation, whether or not maximum-likelihood techniques are used, would be to make estimates disregarding the inequalities in **20** and then "round off" as necessary to satisfy them. For example, if an estimate of -1.5 were obtained for some parameter, it would be arbitrarily rounded up to 0. Although this method has been used by others,[6] we consider it unsatisfactory and only use it occasionally as the first step in an iterative calculation of likelihood estimates.

We shall discuss other theoretical aspects of the parameter estimation problem as the need arises in treating particular cases. This section is intended only to lay the theoretical foundation; the following section will give some numerical examples of estimation.

Illustrative Estimation Examples

Our first example is intended to illustrate the likelihood estimation method for a simple case. Assume that the experimental data are:

$$r_1 = 2, \qquad r_2 = 1, \qquad r_3 = 2,$$
$$s_1 = 1, \qquad s_2 = 1, \qquad s_3 = 2.$$

Assume further, that $p_1(1) = p_2(2) = p_3(3) = \frac{1}{3}$. In this case, from Equation **19**, we have the following likelihood function:

$$p_{121} = \left(\frac{1 - w}{2}\right)\left[\frac{1 + 3w^2}{2} + (1 - 3w)(1 - c)\right].$$

To find the largest value of p_{121} we consider:

$$0 = \frac{\partial p_{121}}{\partial w} = \frac{-9}{4}(w - 1)^2 + c(2 - 3w),$$

$$0 = \frac{\partial p_{121}}{\partial c} = \frac{1}{2}(w - 1)(1 - 3w).$$

Consequently, relative extrema exist when either:

$$(w = 1, c = 0) \qquad \text{or} \qquad (w = 1/3, c = 1).$$

The latter values yield the larger likelihood, namely:

$$p_{121}(w = 1/3, c = 1) = 2/9.$$

The restrictions of **20** further require:

$$b + c \leq 1, \qquad b \geq 0, \quad \text{so} \quad b = 0.$$

Finally, since $a = 3w - 2b$, we have as our tentative maximum-likelihood estimates:

$$\hat{a} = 1, \qquad \hat{b} = 0, \qquad \hat{c} = 1.$$

However, as we shall now see, these values are not the final maximum-likelihood estimates; the correct values are determined by checking the likelihood function values on all the boundaries.

The boundaries of our parameter region, as required by **20**, are:

$$a = 0, \qquad b = 0, \qquad c = 0;$$

$$a = 1, \qquad b = 0, \qquad c = 0;$$

$$a = 0, \qquad b + c = 1;$$

$$a = 1, \qquad b + c = 1.$$

We consider these four cases in turn. In the first two cases, since $w = (1/3)(a + 2b)$, we have

$$p_{121} = 3/4, 2/9.$$

In the third case, using $1 - c = b = (3w/2)$, we have:

$$p_{121} = (1/4)(6w^3 - 9w^2 + 2w + 1).$$

This takes on the value $1/4$ for $w = 0$ and $w = 2/3$, the boundaries on w when $a = 0$, and the interior maximum occurs when:

$$0 = \frac{d(p_{121})}{dw} = (1/2)(9w^2 - 9w + 1) \qquad \text{or} \qquad w = \frac{3 - \sqrt{5}}{6}.$$

It is easily verified that $p_{121} < 3/4$ when $w = (3 - \sqrt{5})/6$. A similar analysis of the fourth case also yields a value of p_{121} less than $3/4$. Thus, the maximum-likelihood estimates are $\hat{a} = \hat{b} = \hat{c} = 0$. Exactly similar calculations, for the

outcome patterns ($s_1 = s_2$) and three responses in a single trial, yield the following maximum-likelihood estimates in the five cases.

$$
\begin{array}{llll}
p_{111} = 1 & \text{for } \hat{a} = 1, & \hat{b} = 1, & \hat{c} = 0. \\
p_{112} = 4/9 & \text{for } \hat{a} = 0, & \hat{b} = 1, & \hat{c} = 0. \\
p_{121} = 3/4 & \text{for } \hat{a} = 0, & \hat{b} = 0, & \hat{c} = 0. \\
p_{122} = \sqrt{3}/6 & \text{for } \hat{a} = 0, & \hat{b} = \sqrt{3}/2, & \hat{c} = 0. \\
p_{123} = 3/4 & \text{for } \hat{a} = 0, & \hat{b} = 0, & \hat{c} = 1.
\end{array}
$$

It is apparent, from these sample calculations, that explicit solutions for maximum-likelihood estimates could be obtained in a straightforward but very tedious manner for trials with more than three responses.

We shall now illustrate the use of our estimation methods by treating two synthetic examples, one of which consists of many repeated trials of three responses each, and the other of which consists of one trial with 75 responses. The examples are "synthetic" because the data for them were obtained by choosing arbitrary values for the behavioral and state parameters, fixing also upon an arbitrary choice of outcome sequences, and then calculating responses that would result by using a table of random numbers. Since this technique of using synthetic data is employed frequently in our work, we shall digress for a moment to show how this is done.

Consider an example of a synthetic trial having four successive responses, the first two of which have the same outcome and the third a different outcome. Our preliminary data are:

$$
\begin{gathered}
m = 3; \quad n = 2; \quad s_1 = s_2 = 1, \quad s_3 = 2; \\
p_1(1) = p_2(1) = p_3(1) = 1/3; \quad a^1 = 0.11, \quad b^1 = 0.01, \\
c^1 = 0.40, \quad a^2 = 0.19, \quad b^2 = 0.01, \quad c^2 = 0.50; \quad N = 4.
\end{gathered} \tag{22}
$$

We next choose four from a table of random probabilities, say:

$$R_1 = 0.862, \quad R_2 = 0.283, \quad R_3 = 0.622, \quad R_4 = 0.461.$$

The first response is, therefore, $r_1 = 3$ since $p_1(1) + p_2(1) < R_1$. The probability vector, after the first response, is now:

$$
p(2) = M^{31}p(1) = \begin{vmatrix} 0.40 & 0.59 & 0.445 \\ 0.59 & 0.40 & 0.445 \\ 0.01 & 0.01 & 0.11 \end{vmatrix} \begin{vmatrix} 0.33 \\ 0.33 \\ 0.34 \end{vmatrix} = \begin{vmatrix} 0.478 \\ 0.478 \\ 0.044 \end{vmatrix}.
$$

Consequently, $r_2 = 1$ since $R_2 \leq p_1(2)$. The new probability vector, after the second response and outcome is:

$$
p(3) = M^{11}p(2) = \begin{vmatrix} 0.11 & 0.01 & 0.01 \\ 0.445 & 0.40 & 0.59 \\ 0.445 & 0.59 & 0.40 \end{vmatrix} \begin{vmatrix} 0.478 \\ 0.478 \\ 0.044 \end{vmatrix} = \begin{vmatrix} 0.058 \\ 0.430 \\ 0.512 \end{vmatrix}.
$$

Again, since $p_1(3) + p_2(3) < R_3$ then $r_3 = 3$. Continuing:

$$p(4) = M^{23}p(3) = \begin{vmatrix} 0.50 & 0.49 & 0.405 \\ 0.49 & 0.50 & 0.405 \\ 0.01 & 0.01 & 0.19 \end{vmatrix} \begin{vmatrix} 0.058 \\ 0.430 \\ 0.512 \end{vmatrix} = \begin{vmatrix} 0.447 \\ 0.451 \\ 0.102 \end{vmatrix},$$

and, since $p_1(4) \leq R_4 < p_1(4) + p_2(4)$, we have finally $r_4 = 2$. This illustrates the method of generating synthetic trials.

An IBM 704 Computer was used to generate 600 synthetic trials of three responses each with the following preliminary data:

$$m = 3; \quad n = 2; \quad s_1 = s_2 = 1; \quad p_1(1) = p_2(1) = p_3(1) = \tfrac{1}{3};$$
$$a^1 = 0.05, \quad b^1 = 0.04, \quad c^1 = 0.50, \quad a^2 = 0.13,$$
$$b^2 = 0.01, \quad c^2 = 0.60; \quad N = 3, \quad q = 600.$$

When the 600 sequences were enumerated the results were as follows:

$$F_{111} = 1, \quad F_{112} = 24, \quad F_{121} = 262, \quad F_{122} = 16, \quad F_{123} = 297,$$

or, equivalently:

$$f_{111} = .00166, \quad f_{112} = .04, \quad f_{121} = .43667, \quad f_{122} = .02667, \quad f_{123} = .495.$$

When these values for f_{ijk} are substituted in **21** we obtain:

$$\hat{a} = -0.0216, \quad \hat{b} = 0.0733, \quad \hat{c} = 0.5329.$$

This illustrates the fact that the estimates yielded by **21** cannot always be used without modification because, as in this case, they may violate the restrictions of **3**. Of course, in this example, the true maximum-likelihood estimates would be the values of a, b, c that maximize the quantity.

$$ln\, L(a, b, c) = ln\, p_{111} + 24\, ln\, p_{112} + 262\, ln\, p_{121} + 16\, ln\, p_{122} + 297\, ln\, p_{123}, \quad \textbf{(23)}$$

where the p_{ijk} are as given by **19** and a, b, c are subject to the restrictions of **3**. These estimates are $\hat{a} = 0.0135$, $\hat{b} = 0.0483$, $\hat{c} = 0.5454$, for which $ln\, L(\hat{a}, \hat{b}, \hat{c}) = -0.94655$.

The actual 75 responses (r_t) for a synthetic trial whose parameters were those of **22**, produced by an IBM 704 run, are as follows:

31321	32213	21213	23121	31233	21231	23231	31232
32131	31332	32321	31232	32332	12121	23231	

(24)

The sequence of outcomes (s_t) that was used in this synthetic trial was as follows:

11212	12121	12211	11112	22111	11211	22111	11111
11211	21211	12221	21111	11211	12211	12111	

(25)

In principle, but not in practice, one can easily find the maximum-likelihood estimates that follow from these values of $p(1)$, r_t, and s_t by maximizing L

in **8**. We shall now describe and illustrate a method that yields approximate values for such estimates.

Approximations good to one decimal place, for each of the six parameters, could be obtained by computing the value of the likelihood function **8** for each of the 11^6 possible sets of parameter values between 0 and 1; actually, there are somewhat less than 11^6 possible sets because of the restrictions that $b^1 + c^1 \leq 1$ and $b^2 + c^2 \leq 1$. Then the set of parameter values yielding the largest likelihood value would be chosen as the approximate maximum likelihood estimated parameters. The IBM 704 code we have used for such calculations computes these values at the rate of about 11/sec., for the case represented by the data of **24** and **25**, including the successive choice of sets of parameter values and selection and printout of the largest likelihood value found for all sets. At this rate of calculation, it would require some thirty hours of IBM 704 time to determine the six parameter values accurately to about one decimal place, for a single trial consisting of 75 successive responses among three alternatives; obviously, some more economical scheme of calculation must be found than simply computing and comparing among all points of a fine lattice in the parameter space.

Two schemes have been used for handling data like that of **23** and **24**. One consists in computing a relatively coarse lattice, perhaps starting with 3^6 points, and then computing a somewhat finer lattice centered around the best point found previously, until all points of the lattice give about the same likelihood value. Another method consists in searching directly for the best parameter values by some computational scheme that tries a point, or points, and then moves next to a new trial point according to some systematic criterion. We shall call the first method the "successive lattice method," and the second method the "direct search method." We shall now present some results obtained using a successive lattice method.

An IBM 704 code was written to make possible a lattice computation over a subspace of the 6-dimensional unit cube that represents the parameter space. This "lattice subspace," and the fineness of the "mesh" within it, is chosen by the experimenter; roughly speaking, the lattice subspace can include any closed interval for each of the six parameters, and the fineness of the mesh can be chosen arbitrarily and independently for each parameter. The following example, for data generated synthetically as for **24** and **25**, will illustrate this procedure.

For our present synthetic trial we chose:

$$m = 3; \quad n = 2; \quad s_t \text{ as in } \mathbf{25}; \quad p_1(1) = p_2(1) = p_3(1) = \tfrac{1}{3}; \quad a^1 = .01,$$
$$b^1 = 0, \quad c^1 = .4, \quad a^2 = .19, \quad b^2 = 0, \quad c^2 = .25; \quad N = 75.$$

The sequence of responses (r_t) produced synthetically by the IBM 704 run were:

$$\begin{matrix} 21312 & 13231 & 32312 & 12323 & 13131 & 21221 & 21323 & 13232 \\ 32323 & 23232 & 31323 & 22313 & 21213 & 23321 & 23232 & \end{matrix} \tag{26}$$

The first lattice, FIGURE 1, was used here. Thus, each parameter was allowed

to take on the values 0, ½, and 1 until all allowable combinations were computed. The result was:

$$ln\ L(0, 0, 0; \tfrac{1}{2}, 0, 0) = 75\ (-0.7285) \geq ln\ L(a^1, b^1, c^1, a^2, b^2, c^2).$$

Following this, the lattice in FIGURE 2 was used. The best result was:

$$ln\ L(0, 0, 0; \tfrac{1}{4}, 0, 0) = 75\ (-0.7207).$$

FIGURES 3, 4, and 5, with three additional lattices and their best results, were then used.

The example was not carried beyond this stage, but it illustrates not only the difficulty in using the lattice method but also its usefulness. Thus, after computing for only about 3600 points (ten minutes of IBM 704 time), we have some idea of the location of the likelihood estimates and also we see that the value of the likelihood function varies little as the parameter values are changed rather substantially. We shall now turn to the direct search type of method.

	a^1	b^1	c^1	a^2	b^2	c^2
Min.	0	0	0	0	0	0
Max.	1	1	1	1	1	1
Mesh	2	2	2	2	2	2

FIGURE 1.

	a^1	b^1	c^1	a^2	b^2	c^2
Min.	0	0	0	0.25	0	0
Max.	0.5	0.5	0.5	0.75	0.5	0.5
Mesh	2	2	2	2	2	2

FIGURE 2.

There are a number of direct search methods that could be used. The steepest ascent methods are a popular variety for this type of problem. One of these methods simply starts at some more or less arbitrary point in the parameter space, then computes the value of the function on some fairly small lattice centered on this point, then determines from these points the direction in the parameter space that ascends most rapidly, then moves step by step in this direction until progress upward ceases, and then repeats this general process until a relative maximum seems to have been reached. Unfortunately, the method of steepest ascent does not seem to work well in our type of problem, where the function is a polynomial of very high degree in six parameters, so that there is no assurance that a relative maximum is also a general maximum. We favor, instead, a method like that of Hooke and Jeeves,[11] in which the ascent is more direct and is done without seeking the most rapid ascent at each move. Their method was coded for the IBM 704, and we have modified this code for our present purposes (*Appendix B*). Its use will be illustrated and discussed in later sections, both for seeking maxima in cases like those just treated by the lattice method and in cases like those represented by **23**.

We turn next to a few questions that arise in considering the statistical significance of some of the results obtained by using maximum-likelihood estimates derived in the manner just described.

Parameter Significance

In the preceding section, two sets of synthetic trials were analyzed to estimate the parameter values that produced them. In both instances, the estimates were apparently not very close to the true values. Our present task is to develop a significance test that will indicate whether or not such discrepan-

	a^1	b^1	c^1	a^2	b^2	c^2
Min.	0	0	0	0.125	0	0
Max.	0.25	0.25	0.25	0.375	0.25	0.25
Mesh	2	2	2	2	2	2

$ln\ L\ (0, 0, 0; 0.375, 0, 0) = 75\ (-0.7191)$

FIGURE 3.

	a^1	b^1	c^1	a^2	b^2	c^2
Min.	0	0	0	0.31	0	0
Max.	0.12	0.12	0.12	0.43	0.12	0.12
Mesh	2	2	2	2	2	2

$ln\ L\ (0, 0, 0; 0.31, 0, 0) = 75\ (-0.7184)$

FIGURE 4.

	a^1	b^1	c^1	a^2	b^2	c^2
Min.	0.001	0.001	0.001	0.28	0.001	0.001
Max.	0.061	0.061	0.061	0.34	0.061	0.061
Mesh	2	2	2	2	2	2

$ln\ L\ (0.001, 0.001, 0.31; 0.31, 0.001, 0.001) = 75\ (-0.7190)$

FIGURE 5.

cies may reasonably be attributed to random fluctuations in our experiments. Such tests will also show how confidence intervals may be chosen for parameter estimates obtained in this manner.

In one case, that of 600 trials each of three responses, the true values and the estimates were as shown in TABLE 1. The first significance test is directed toward a comparison of the observed and theoretical distributions, as shown in TABLE 2.

From the data of TABLE 2 we find $\chi^2 = 5.12$, indicating a degree of discrepancy between theoretical and observed distributions that would arise less than 10 per cent of the time; actually about as large a discrepancy as we would want to tolerate routinely. From a confidence region standpoint, we would

therefore say that a set of parameter values that led to a distribution with no larger discrepancy than this from the observed would be included within a confidence region based on the probability level corresponding to $\chi^2 = 5.12$. In particular, we would want our parameter estimates (0, 0.07, 0.53) to satisfy this criterion, and we now verify that this is a point within our confidence region; the new "theoretical" values in TABLE 3 are based on this set of parameter values, rather than upon the true values used in the synthetic run. From the data of TABLE 3 we find $\chi^2 = 1.7$, indicating that our estimates are well within the critical region determined by our data. Here then is one general method for determining points within the critical region.

TABLE 1
TRUE VALUES AND ESTIMATES

	a^1	b^1	c^1	a^2	b^2	c^2
True	0.05	0.04	0.50	0.13	0.01	0.60
Est.*	0	0.07	0.53		None	

* These are not really maximum-likelihood estimates.

TABLE 2
OBSERVED AND THEORETICAL DISTRIBUTIONS

	p_{111}	p_{112}	p_{121}	p_{122}	p_{123}
Theoretical*	0.00175	0.04160	0.44900	0.04300	0.46500
Observed	0.00167	0.04000	0.43667	0.02667	0.49500

* Based on the true parameter values.

TABLE 3
THEORETICAL AND OBSERVED PARAMETER VALUES

	p_{111}	p_{112}	p_{121}	p_{122}	p_{123}
Theoretical	0.003	0.044	0.433	0.035	0.487
Observed	0.00167	0.04000	0.43667	0.02667	0.49500

It is already clear that a run of 600 trials does not narrow the critical region down to two-decimal accuracy for our parameters in this particular case. We turn now to our second type of estimate, that based upon a run of one trial with 75 responses among the three alternatives, as displayed in **26**.

We have already noted that:

$$ln\ L(0, 0, 0; 0.31, 0, 0) = 75\ (-0.7184) = ln\ L(\text{est. parameters}),$$

is the largest log-likelihood value obtained from the sequence of lattices used in approximating the maximum-likelihood estimates in our 75-response trial. A calculation made with the IBM 704 shows also that:

$$ln\ L(0.01, 0, 0.4; 0.19, 0, 0.25) = 75\ (-0.7595) = ln\ L(\text{true parameters}).$$

We consider the likelihood-ratio:

$$2\ ln\ \lambda \equiv 2\ ln \left[\frac{L(\text{est.})}{L(\text{true})}\right] = 6.165 = \chi^2.$$

This value of χ^2 is well within our confidence region, taking into account the six degrees of freedom represented by the six parameters.

In general, then, we will consider a set of parameter values θ to be within our confidence region, relative to a maximum-likelihood estimate denoted $\hat{\theta}$, provided the following condition is satisfied:

$$2\ ln\ \lambda = 2[ln\ L(\theta) - ln\ L(\hat{\theta})] = \chi^2 \leq \chi_0^2,$$

where χ_0^2 is chosen so that the probability of observing no larger value is 0.95 if $\hat{\theta}$ were correct, if we work at the 5 per cent confidence level. For example, if an experimental trial with one animal yields maximum-likelihood estimates $\hat{\theta}$, and the likelihood value for another animal's data, computed using the parameter value $\hat{\theta}$, is L^*, then we will not consider the two animals to differ (on the basis of this experimental data) unless

$$2[ln\ L^* - ln\ L(\hat{\theta})] > \chi_0^2.$$

In other words, if we assume that $\hat{\theta}$ estimates the true value common to the two animals, then this test will reject the hypothesis only if the second animal's experimental data yield a likelihood value incompatible with this hypothesis.

An example will illustrate this principle. Two rats were run in a three-choice experiment with the following results:

Rat 1: $r_t =$	13232	13132	32132	12132	12321	32121	31321	31312
	12213	21211	21213	13132	13131	23232	1313	
$s_t =$	11212	11112	11111	11111	11111	11111	11111	11111
	11212	11111	11111	11111	11111	11111	1111	
Rat 2: $r_t =$	13213	13223	21321	32321	32313	21311	21311	32131
	32211	31313	21313	21313	31313	23132	13132	
$s_t =$	11212	12121	12211	11112	22111	11211	22111	11111
	11211	21211	12221	21111	11211	12211	12111	

After several IBM 704 mesh runs, the following approximate values were found for the six parameters for Rats 1 and 2:

$$\textit{Rat 1:}\ (0.06, 0, 0.4; 0.13, 0, 0.6) = \hat{\theta}_1.$$

$$\textit{Rat 2:}\ (0.09, 0, 0.5; 0.13, 0, 0.4) = \hat{\theta}_2.$$

If we take $\chi_0^2 = 12.6$, corresponding to a 5 per cent confidence level based on six degrees of freedom, then the confidence region for Rat 2 will include any set of parameters θ such that $2[ln\ L(\hat{\theta}_2) - ln\ L(\theta)] \leq 12.6$; in particular, this condition will test whether Rat 1 is distinguished from Rat 2 when we use $\theta = \hat{\theta}_1$.

In this case, we found $[2\ ln\ L(\hat{\theta}_2)] = -140.46$, so our condition becomes $[-ln\ L(\theta) \leq 76.53]$. Actually, $-ln\ (\hat{\theta}_1) = 73.09$, so that Rat 1 is indistinguishable from Rat 2 on this basis.

We also found several sets of parameter values within the confidence region, so defined, for Rat 2. Among these are:

$$b^1 = 0, \qquad a^2 = 0.13, \qquad b^2 = 0, \qquad c^2 = 0.4,$$

and

$$a^1\text{: } 0.08,\ 0.09,\ 0.08,\ 0.07,\ 0.09,\ 0.08,\ 0.07,\ 0.06,\ 0.05,\ 0.04;$$

$$c^1\text{: } 0.5,\ \ 0.4,\ \ 0.4,\ \ 0.4,\ \ 0.3,\ \ 0.3,\ \ 0.3,\ \ 0.3,\ \ 0.3,\ \ 0.3.$$

Two sets outside the confidence region are:

$$\left.\begin{array}{l} a^1 = 0.03,\ 0.02 \\ c^1 = 0.3,\ 0.3 \end{array}\right\} \quad \text{and} \quad b^1 = 0,\ a^2 = 0.13,\ b^2 = 0,\ c^2 = 0.4.$$

On another trial, for Rat 2 with 51 responses, the parameter estimates were as follows:

$$\hat{\theta} = (0.06, 0, 0.3; 0.13, 0, 0.6) \quad \text{with} \quad -2\ ln\ (\hat{\theta}) = 83.844.$$

These parameter values seem very close to the ones listed above for Rat 1 so, on the basis of these experimental data, the two rats would not seem to be distinguishable. When these parameter estimates $(\hat{\theta})$ for Rat 2 are used to determine the likelihood based on the previous 75-response trial for Rat 2, we find $-ln\ (\hat{\theta}) = 74.37 \leq 76.53$, so that again we would not distinguish any difference in Rat 2 between these two trials.

The examples just discussed should suffice to illustrate our methods for estimating parameters, and for assessing the significance of differences among them. Basically, it is familiar maximum-likelihood estimation with confidence regions based on the likelihood-ratio significance test.

Preliminary Experimental Results

Some preliminary experimental results have been obtained with two rats, here called Rat *I* and Rat *J*. Unfortunately, Rat *J* died before the planned schedule of experiments had been completed. The full schedule has also not yet been completed for Rat *I*.

Two types of experiments were conducted. In one type (Schedule A), a probability of reinforcement was assigned to each alternative, and the number of reinforced responses depended upon the actual responses made by the animal. In the other type (Schedule B), the sequence of reinforcements was entirely independent of the actual responses made.

Schedule A used the probabilities (0.2, 0.6, 0.8) or (0, 0.6, 1). Schedule B normally consisted of repeated blocks of 12 stimuli, each block starting with six reinforced responses and ending with six unreinforced responses. Normally, an animal was kept responding for about one hour and made 100 to 350 responses in that period.

The animal could press any one of six bars at each response. If the animal moved to the center of the cage so as to activate the photocell, after pressing any bar, then all six bars were reset and made active. The apparatus recorded the time at which each bar was pressed and the time when the photocell was first activated after each bar-press. For the present analysis, the responses used were the bars first pressed after each resetting by the photocell; corresponding to each such response r_t, there is either an outcome $s_t = 1$, denoting that the response was reinforced, or an outcome $s_t = 2$, denoting that the response was not reinforced. Also, for the present analysis, bars symmetrically opposite within the cage were treated as though they were identical; thus, there were three alternatives available for each response: a left bar, a middle bar, and a right bar.

Every effort was made to keep the opposite sides of the cage as nearly identical as possible. For example, the walls behind the bars were painted differently for left, middle, and right, but the same for each direction on both sides of the cage. Nevertheless, it became apparent that the experimental animals

TABLE 4
MAXIMUM-LIKELIHOOD ESTIMATES
Starting State Assumed

Trial No.	Rat	$\hat{a}^1$	$\hat{b}^1$	$\hat{c}^1$	$\hat{a}^2$	$\hat{b}^2$	$\hat{c}^2$	N	L^*	d
144	J	0.999	0.085	0.915	0.949	0.007	0.987	231	0.772	0.036
146	J	0.949	0.101	0.899	0.949	0.151	0.849	134	1.017	0.063
148	J	0.999	0.063	0.931	0.999	0.033	0.967	269	0.829	0.031
150	J	0.899	0.100	0.900	0.999	0.100	0.700	144	1.064	0.058
196†	I	0.999	0.001	0.998	0.999	0.050	0.949	206	0.944	0.041

† Search calculation not complete.

behaved differently on opposite sides of the cage, so grouping together two bars symmetrically placed in the cage proved to be a more arbitrary combination of response classes than had been intended. Consequently, the present analysis is offered as an example of results obtained with a somewhat arbitrary grouping of response classes, rather than for three response classes that might more reasonably be expected to satisfy the symmetry conditions upon which our mathematical model is based.

The results shown in TABLE 4 were obtained using Schedule A (0.2, 0.6, 0.8) with Rat J, and using Schedule B with Rat I. The quantity d, in TABLE 4, is the relevant value for χ_0^2 working at the 1 per cent level with six parameters; that is, any set of six parameters yielding a likelihood value less than $(L^* + d)$ is within the 1 per cent confidence region. (In the table, $L^* = [-ln\ L]/N$.) For example, on Trial 148, the following six sets of parameter values include three sets that are within this confidence region ($L^* \leq 0.86$) and three sets that are not.

The first three sets of parameter values shown in TABLE 5 differ from the maximum-likelihood estimates, shown in TABLE 4, by the following maximum amounts: 0.050, 0.100, 0.131, 0.100, 0.072, and 0.072. It is likely that parame-

ter values could be found within the confidence region that would show even greater deviations. On the other hand, the second three sets of parameter values shown in TABLE 5 differ from the maximum-likelihood estimates, shown in TABLE 4, by the following minimum amounts: 0, 0.013, 0.031, 0, 0.018, and 0.032.

Thus only rather slight changes in certain parameter values will sometimes place the set outside the confidence region. It can only be concluded, from the data of TABLE 4, that the maximum-likelihood estimates are reasonably similar from trial to trial and from rat to rat; a more extensive experiment and analysis is required in order to test this apparent stability of parameter estimates and to narrow the confidence regions appreciably.

TABLE 5
TRIAL 148 LIKELIHOODS

a^1	b^1	c^1	a^2	b^2	c^2	L^*
0.949	0.100	0.900	0.999	0.052	0.948	0.860
1.000	0.200	0.800	0.899	0.105	0.895	0.858
0.999	0.100	0.900	0.999	0.001	0.898	0.848
0.999	0.100	0.900	0.999	0.001	0.999	0.920
0.999	0.100	0.700	0.898	0.102	0.898	0.912
0.999	0.050	0.900	0.999	0.051	0.898	0.878

TABLE 6
MAXIMUM-LIKELIHOOD ESTIMATES
Starting State Predicted

Trial No.	$p\hat{L}(1)$	$p\hat{C}(1)$	$p\hat{R}(1)$	$\hat{a}^1$	$\hat{b}^1$	$\hat{c}^1$	$\hat{a}^2$	$\hat{b}^2$	$\hat{c}^2$	L^*
146	0.215	0.739	0.046	0.937	0.112	0.875	0.987	0.086	0.890	1.010
148	0.331	0.562	0.107	0.993	0.070	0.924	0.999	0.026	0.974	0.822
150	0.080	0.356	0.564	0.942	0.076	0.918	0.999	0.063	0.880	1.056

The maximum-likelihood estimates, shown in TABLE 4, were all computed on the assumption that the first response of each trial was equally likely to be any one of the three bars, thus: $p_1(1) = p_2(1) = p_3(1) = \frac{1}{3}$. The computations were done on the IBM 704, using the modified search code described in *Appendix B*.

Another calculation was made, also using the search code, in which the initial state for each trial was taken to be the ending state for the preceding trial; this is an alternative to the assumption of equal probability among bars at the start of each trial. The results are shown in TABLE 6. It is clear, from TABLE 6, that the use of predicted starting states improves the fit of the model to the data, since each value of L^* is smaller than in TABLE 4, where the starting state was assumed to be probability $\frac{1}{3}$ for each bar. Unfortunately, the estimates of TABLES 4 and 6 are not entirely reliable because the search code does not absolutely ensure correct likelihood values.

The starting states shown in TABLE 6, for each trial, were calculated from the data for the preceding trial. More specifically, the starting state for Trial 144 was taken arbitrarily to be $p_1(1) = p_2(1) = p_3(1) = 1/3$. This yielded the maximum-likelihood estimates for Trial 144, as shown in TABLE 4. The end state for Trial 144, based upon the estimated parameters, was calculated to be: $p_L(232) = 0.215$, $p_C(232) = 0.739$, $p_R(232) = 0.046$. Since the first response in Trial 146 is the next one made by Rat J after the 231st response in Trial 144, with a rest of one hour intervening, the end state of Trial 144 was taken as the starting state for Trial 146. Similar reasoning applies to Trials 148 and 150.

The actual schedule of reinforcements used on Trials 144, 146, 148, and 150 led to the gross results shown in TABLE 7. The results in TABLE 7 followed from

TABLE 7
DISTRIBUTION OF BAR PRESSES

Trial No.	Number of times bar pressed						
	Total	Reinforced			Unreinforced		
		Left	Center	Right	Left	Center	Right
144	231	36	128	2	24	31	10
146	134	12	36	9	40	28	9
148	269	3	97	60	29	67	13
150	144	38	18	20	10	48	10

TABLE 8
COMPARISON OF ACTUAL AND SCHEDULED REINFORCEMENT PROBABILITIES

Trial No.	Scheduled			Actual		
	L	C	R	L	C	R
144	0.6	0.8	0.2	0.60	0.81	0.17
146	0.2	0.6	0.8	0.23	0.56	0.50
148	0.2	0.6	0.8	0.09	0.59	0.82
150	0.8	0.2	0.6	0.79	0.27	0.67

use of Schedule A (0.2, 0.6, 0.8), with a permutation of the probabilities among the bars after Trials 144 and 148. The actual relative frequencies are compared with the scheduled probabilities in TABLE 8. The starting states shown in TABLE 6 clearly show the tendency for the rat to select the responses that have led to reinforcements most regularly during the recent past, but they also show that former good alternatives continue to be selected for a considerable period of time.

It is interesting that $(\hat{b}^1 + \hat{c}^1)$ and $(\hat{b}^2 + \hat{c}^2)$ are close to unity in value for each trial shown in TABLE 6. Furthermore, if the combining-of-classes assumption holds, then necessarily

$$C^s \equiv \frac{1 - a^s}{2} - (1 - b^s - c^s) = 0 \text{ for } s = 1, 2.$$

The values for C^s actually observed are given in TABLE 9. Although the observed values for C^s are all near zero, our experiments and analysis are not extensive enough to determine whether or not the combining-of-classes assumption is acceptable for this test situation. This question is being investigated further.

It now seems that one trial, with $N = 1000$, is about right to yield parameter estimates accurate to two decimal places. With our present computational methods, it requires about 30 min. of IBM 704 time to obtain two separate sets of parameter estimates for one such trial, using the search code described in *Appendix B*. For such a trial, the confidence region, at the 1 per cent level, has a value of $d < 0.01$; this seems to be precise enough to yield reasonably sensitive comparisons between sets of parameter estimates.

The effect of small changes in parameter values, in the general neighborhood of values observed for Rats *I* and *J*, is being investigated by the use of synthetic runs on the IBM 704. This synthetic technique is being used to explore empirically the distribution of *L*, and to explore changes induced by changes in the experimental situation. In other words, the synthetic rat also becomes a subject of experimental investigation and various comparisons between gross

TABLE 9
OBSERVED VALUES FOR C^s

Trial No.	144	146	148	150	196
C^1	0	0.018	−0.003	0.023	0
C^2	−0.019	−0.018	0	−0.007	0

behavior of synthetic rat and actual rat should help to highlight similarities and differences between a real rat and its mathematical model.

Conclusions

(1) Stimulation through electrodes implanted in the hypothalamic region of a rat's brain can be used efficiently to provide reinforcing stimuli in a learning experiment extending over a period of months.

(2) Maximum-likelihood estimates of the six parameters in the symmetry model can be approximated, using a search code, in less than 30 min. of IBM 704 time, for a trial consisting of 1000 successive responses from among three alternatives.

(3) Estimated values of the six symmetry model parameters are reasonably constant from trial to trial for a single rat, when measured in an experiment in which the rat has three alternatives for each response and the schedule of reinforcement depends upon the responses actually made.

(4) The very limited experimental data analyzed are compatible with the hypothesis that estimated parameter values are the same for the two rats tested, even though the parameters are measured using two quite different types of reinforcement schedules.

(5) The data analyzed are also compatible with the hypothesis that the combining-of-classes condition is satisfied.

Acknowledgments

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The central concept for the experiment was discussed with R. R. Bush, Eugene Galanter, and R. D. Luce in the summer of 1958; they very generously provided information regarding their current work and the recent work of others that helped in planning the experiment and analysis. Similar more recent conversations with Patrick Suppes and Richard C. Atkinson, and with J. David Birch, have helped in choosing among experimental possibilities.

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I am especially grateful to Fred E. Horvath, who supervised the design and construction of the experimental apparatus, implanted the electrodes, trained and ran the experimental animals, and made the experimental trials go smoothly. Above all, I am indebted to him for many pointers regarding the psychology of rats and their behavior in learning experiments of this kind. A summary of his report on the experimental apparatus and techniques is included here as *Appendix A*; a more detailed report will be published elsewhere.

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Appendix A

Experimental Procedure

Subjects and electrode implantation. The majority of the experimental work was carried out with two male, mature albino rats. The electrodes and implantation techniques used were those describe in detail by Olds and Milner.[12] In brief, the electrodes are a pair of twisted silver wires, diamel coated for insulation except at their tips, thus restricting the stimulus to that point. Each pair of electrodes is secured to a lucite pedestal that, in turn, is held in place on the skull by screws through the skull bone. This technique has proven to be very satisfactory for longitudinal studies using implanted rats.

The area selected for implantation was the medial forebrain bundle, lateral to the anterior hypothalamus, an area Olds has rated as positively reinforcing.[13]

Experimental apparatus and procedure. The reinforcing stimulus was an AC pulse of 10 to 25 μamp. intensity, with duration a function of the length of time the bar was depressed, up to an automatic cut-off time of 0.6 sec. Stimulus threshold values for reinforcement in this area are about 12 μamp., while currents in excess of 25 μamp. are best avoided since they make the rats hyperexcitable, disrupting their performance in a choice situation.

Following a one-week postoperative recovery period, the rats first were trained to bar press for a stimulus in an enclosed box with a single bar. The response was shaped up in the usual manner of giving reinforcements for successively closer approximations of the desired response, until the bar finally is depressed. Thereafter reinforcements are given automatically after each bar press, a response both rats were making at rates of 500 to 1000/hour within a few hours of training.

The testing of rats in a choice situation necessitates the rat's return to a neutral choice point between responses. There are advantages to having them proceed to a choice point on their own, so the rats next were trained to go to the opposite side of the box after every bar press. This more complex response also was shaped up by the method of reinforcing successive approximations to the required response sequence. When they had learned to make this total response with a minimum of errors (usually within 10 hours of experimental sessions), they were advanced to the final training phase.

The first apparatus used in the final phase was a large circular cage with three bars equally spaced around its circumference and a disc in the center marking the choice point. Although the rats readily transferred their previous learning to this three-bar condition—going to the choice point after pressing any of the three bars—they also demonstrated a strong bias that practically reduced this to a two-choice situation. Upon reaching the center choice point, they showed

a strong preference for choosing their next response from between the two bars facing them from across the cage, rather than making the inefficient response of turning around to return to the bar just pressed. Reinforcing the bars differentially may have overcome this bias, but it seemed preferable to start with a less biased three-choice situation.

Therefore, a new apparatus was designed to incorporate two desirable features: (1) a choice point at which the rat's visual field would include all of the bars from which he is expected to choose; and (2) ease in converting the apparatus to test the rats on any number of bars. The apparatus is illustrated in FIGURE 6, and consists basically of two fan-shaped center sections, the sides of which can be pivoted at the center point to encompass the desired number of removable U-shaped panels at each end. When the rat passes through the photocell beam at the choice point, the stimulus circuit is reset so he can be

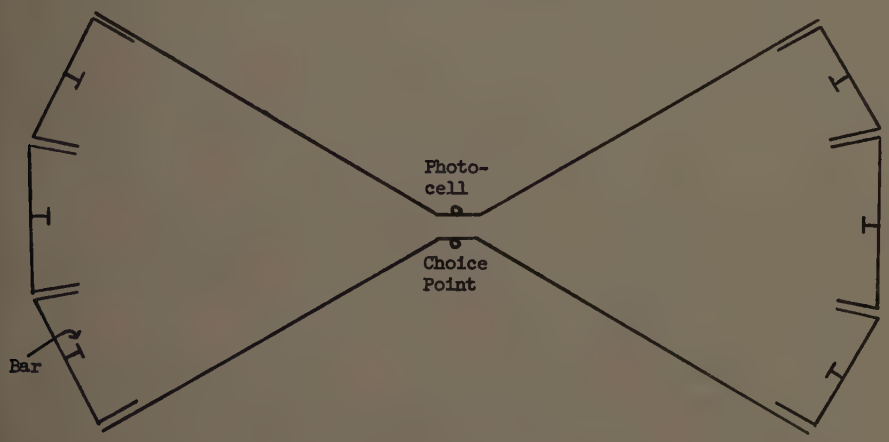

FIGURE 6.

reinforced for his next press of any of the bars. Control circuitry makes it possible to run experiments using patterned or relatively random partial reinforcement schedules, with either the same or differential schedules on each bar.

In the initial sessions in this apparatus, the rats were run with one bar at each end, but they adapted so quickly to this new condition that in subsequent sessions they were run with three bars. Under conditions of 100 per cent reinforcement, there were very few failures to return to the choice point between bar presses. Under partial reinforcement schedules, failures to return increased, indicating emotional outbursts rather than lack of training. The reversal of differential reinforcement schedules has pointed up one problem with this apparatus: a possible nonequivalence of the bars at both ends. Despite our attempts to eliminate all differentiating cues, the rats are discriminating sufficiently between the ends to select different bars at each end. If this nonequivalence proves to be the case, the apparatus will be modified by retaining the bars at one end only. At the other end the bars will be replaced by a blank panel attached to the center section to form a cul-de-sac, with the

photocell relocated close to the end of this blind alley. This will force the rat to enter the alley to reset the device, so that on leaving the alley he can again face all of the bars from which he is expected to choose.

Appendix B

Search Codes

Maximum-likelihood estimation and many other situations require an efficient computational procedure for determining the location of the largest (or smallest) value of a function of several real variables. Some or all of the variables may also be restricted to some region; common restrictions would be nonnegativity or limitation to the unit n-cube.

Our maximum-likelihood estimation problem is essentially that of finding the values for a set of real variables that maximizes the value of a polynomial in the variables, where the variables are restricted to a specified portion of the unit cube.

Mathematically, if we let x denote the set of real variables $(x_1, x_2, \cdots, x_n)$, $f(x)$ a real function of the variables x, and R the domain over which the variables are allowed to range, then our problem is to find a set of values $\hat{x}$ such that $f(\hat{x}) \geq f(x)$ for all x in R. Or, rather, our need is for a computational procedure that will solve this problem economically when $f(x)$ is one of the likelihood functions encountered in analyzing experimental data.

When $f(x)$ is a polynomial, given explicitly as is ours in the likelihood estimation situation, there is no theoretical difficulty in finding the required values $\hat{x}$ by the ordinary methods of calculus. In brief outline, for the case when R is the unit n-cube, all that is necessary is:

(1) Find the roots of the polynomial $f'(x) \equiv df(x)/dx$. For each such root, if it is in R, test to determine whether or not it yields a relative maximum for $f(x)$. Let $M = f(\bar{x})$ be the largest of these relative maxima.

(2) Let $X \equiv (X_1, X_2, \cdots, X_n)$, where X_i is either x_i, 0, or 1. Then $f(X)$ is the value of the function on some bounding hyperplane of the unit n-cube; the dimension of this hyperplane is the number of x_i included in X. By the method outlined in (1), the largest value of $f(X)$ can be determined. This includes the case where all X_i are 0 or 1, representing the values at the corners. Let $M(\bar{X}) = f(\bar{X})$ be the largest of the relative maxima for the bounding hyperplane defined by X.

(3) The required value of $\hat{x}$ is the $\bar{X}$ that satisfies the condition

$$f(\bar{X}) \geq M(\bar{X}) \text{ for all } \bar{X}.$$

In other words, the locations of the largest interior relative maxima are determined on all bounding hyperplanes, and the solution to our problem is the one that yields the largest value of the function. When the degree of the polynomial is high, or the number of variables is large, the techniques of the calculus are generally too inefficient to permit their use, as is normally the case for our likelihood functions. Consequently, we have turned to the approximation method to be described now.

Hooke and Jeeves[11] have developed a procedure for searching through the

domain R, and have used the procedure successfully on various problems. Their procedure involves a combination of two rather different types of searching operations: they have called one type "direct search" and the other type "pattern search." Only the central features of their procedure will be presented here.

Direct search starts with an arbitrary point x^1 in the n-dimensional domain R, where R is defined by the inequalities:

$$d_i \leq x_i \leq D_i \quad \text{for} \quad i = 1, 2, \cdots, n.$$

The first computational step is to compare $f(x^1)$ and $f(x_1^1 + \Delta, x_2^1, \cdots, x_n^1)$, where Δ is some positive real number and where $x_1^1 + \Delta$ is changed to D_1 if $x_1^1 + \Delta > D_1$. If the change in the variable yields a larger value of the function then the new value is retained. If not, x_1^1 is decreased by amount Δ, but not to a value less than d_1, and the changed variable is retained if and only if the function value increases. The procedure is repeated with x_2, then with x_3, and so on through x_n. If no change in the variables yielded an improvement, then the entire process is repeated with $(\Delta/2)$ replacing Δ, unless $(\Delta/2) < \Delta_{\min}$, until the process either terminates or an improved value of the function is located at some stage.

Pattern search follows each instance of success in direct search. If x^o denotes the starting point of a direct search step, and x the improved point at the termination of direct search, then pattern search starts with a direct search from the point $2(\bar{x} - x^o) \equiv \bar{x}^o$; as always, if any coordinate violates the condition

$$d_i \leq 2(\bar{x}_i - x_i^o) \leq D_i$$

then it is replaced by the appropriate value of d_i or D_i. In this direct search, as part of pattern search, the value of $f(x^o)$ is ignored and comparisons are with $f(\bar{x}^o)$ at each step during pattern search; furthermore, the terminal value of $(\Delta/2^g)$ in use at the termination of direct search is used unchanged throughout the direct search portion of pattern search. If direct search from $\bar{x}^o$ yields a terminal point $\bar{\bar{x}}^o$ such that $f(\bar{\bar{x}}^o) > f(\bar{x})$, then a new pattern search starts at the point $2(\bar{\bar{x}}^o - \bar{x})$, and $\bar{\bar{x}}^o$ and $\bar{x}$ become the new values for $\bar{x}$ and x^o in this pattern search step. On the other hand, if $f(\bar{\bar{x}}^o) \leq f(\bar{x})$ then pattern search is terminated and direct search is done about $\bar{x}$ with $(\Delta/2^{g+1})$ replacing $(\Delta/2^g)$.

Eventually the search terminates, when $\Delta(2^{g+1}) < \Delta_{\min}$, and the desired value of $\hat{x}$ is approximated by the final value of x in the search process. This brief description omits several important technical details, and says nothing of the actual computer codes used, for which the reader must be referred to the paper of Hooke and Jeeves.[11]

The search code of Hooke and Jeeves was necessarily modified slightly for our maximum-likelihood problem, because the domain R is not of quite the same form as that treated by Hooke and Jeeves. There are two additional restrictions on the variables, in our case, namely:

$$b^1 + c^1 \leq 1, \qquad b^2 + c^2 \leq 1.$$

Primarily for convenience in modifying the computer code, and because the

change seemed adequate to take proper account of these two added restrictions, the "modified search code" actually used retained the altered value of a variable b (or c) and changed c (or b) as necessary to satisfy the restriction $b + c \leq 1$. For example, if b^1 is to be replaced by $(b^1 + \Delta)$ when $(b^1 + \Delta \leq 1)$ and $(b^1 + \Delta + c^1 > 1)$ then c^1 is replaced by $(1 - b^1 - \Delta)$. The calculations reported upon in this paper, as using a search code, were all done with this simple modification of the Hooke-Jeeves code.

It is evident that the search code does not necessarily and inevitably terminate at a point that is even near the correct one. Nor does it seem likely at present that any computational procedure will soon be found that is both reasonably economical and guaranteed to yield an approximate solution near the correct one. Consequently, all of our empirical results are open to question on this ground.

We have sought protection against this danger of error in several ways. The three principal ways were: (1) comparison of search code approximations with lattice code approximations in sample instances; (2) comparison of search code approximations with others obtained using starting points as distant as possible from the previous solutions; and (3) checks for agreement of solutions by search code techniques with a few simple likelihood estimation problems for which exact solutions are known.

Of course, the most important source of protection against such errors is the comparison of calculated parameters from among independent trials with different animals. The results reported in the present paper were subjected to all of these tests, except for the incomplete calculation of parameters for Rat *J*.

DECISION THEORY APPLICATIONS IN HUMAN DECISION MAKING

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An important aspect of decision theory is its potential as a unifying analytic framework for several levels of human decision making. To introduce the discussion, let us first look briefly at two applications, illustrative of quite different decision-making levels, that sprang largely from research-system needs of the United States Navy.

About two years ago Cron and Martin,[1] at the Underwater Sound Laboratory, New London, Conn., tested a group of about 20 observers in two simple detection tasks, for the purpose of comparing their performance to that of a decision theory ideal observer (which we may call a "generalized" ideal observer). Such an observer is one who performs a likelihood ratio measurement, then compares the result of this measurement with a bias or threshold. Each of the two tasks required a binary decision; one task used a *numerical* presentation and the other, a *graphic* presentation. In both cases, the likelihood ratio measurement consists of taking an average.

The numerical task was given in four parts. In Part 1, nine numbers were drawn at random from a population of normal deviates (the noise population) or from a similar population to which a small constant was added (the signal-plus-noise population). The nine numbers were listed on a card, and the observer was given the card and asked to decide which of the two populations the nine numbers came from. A run consisted of an equal number of cards from each of the two populations arranged in random order, and the observers were told this fact beforehand. Observers were also told that a correct decision was worth $+1$ and an error -1 (the so-called unit cost matrix). The difference between the two populations appeared visually marginal. As a more exact measure, the numerical signal-to-noise ratio was -6 db.

Part 2 of the numerical task was, with two changes, identical to that of Part 1. First, four numbers per sample were used instead of nine, and second, the S/N ratio was increased to -2.5 db. The net effect of these two changes is to leave the index of detectability, d' (in the sense of Tanner and Swets[2]) unchanged from that of Part 1.

The results of these tests are that in Part 2, when only four numbers had to be processed per sample, performance of the human observers was indistinguishable from that of the generalized ideal observer. In Part 1, however, when nine numbers had to be processed, human observer performance was significantly poorer than that of the generalized ideal observer. This result suggests that, at least for the type of test attempted, d' may not be an invariant of observer performance.

Part 3 of the numerical task was identical with the four-number, equal a priori probability task of Part 2, except that cost ratios were altered by a factor of 4 from the unit costs used previously. As a result of this cost change, the ideal observer should use a much lower threshold to decide (S + N) than that

used in Part 2. Actual observers did, in fact, lower their threshold somewhat, but went only part way toward the ideal observer threshold. Interestingly enough, their d' remained the same as that of the ideal observer, that is, their efficiency remained unimpaired.

Part 4 was identical to the four-number task of Part 2 with a unit cost matrix; however, the a priori probability of occurrence of a sample from the noise-alone population and from the signal-plus-noise population was changed. Instead of being equally likely, this probability varied from run to run, up to a ratio of 4:1. As before, the observer was told what a priori probability to expect in any run, as well as costs and other facts of the experiment. The result of the Part 4 test is that the observers did not take a priori probability into account in selecting a threshold; furthermore, their average efficiency decreased: d' went down.

Finally, the same random numbers used in Part 3, above, were plotted as y-coordinates on a graph and connected with straight lines. Results of this visual averaging task were compared with the results of the corresponding numerical task, and no statistically-significant difference was found.

These tests are of the same type as the more basic and comprehensive ones reported by Tanner and Swets (*op. cit.*), and the results quoted above do not appear to be contradictory to theirs. We note, however, the variability of observer performance with constant experimental d', the relative sensitivity to cost variation and insensitivity to a priori probability variation, and the difference in dependence of actual detectability upon cost versus a priori probability, despite their similar theoretical roles.

This experiment is at the psychophysical level of application; let us examine briefly a second level of great practical interest, the real-life operational level.

Here the problem we shall discuss is that of conflicting information or, in psychological terms, incongruent stimuli. I shall comment chiefly on the decision-theoretic model* that is more fully treated in an unpublished paper by Becker and myself.[3]

The problem is to develop a model where a decision must be made, and information comes to the final decision maker from several sources of differing validity. Such problems occur every day; the desired solution is a decision rule more-or-less independent of such particulars as the type of stimuli and the order of the decision. What is proposed is a model containing, as in a standard decision-theory formulation, one signal (or "real world occurrency") space, but that includes two (or more) observation spaces. Each point in a given observation space is obtained from a point in signal space to which is combined a sample from a noise space. The observation spaces may be the decision spaces of inaccurate subdecision makers, who report what they think they see in the signal space to the final decision maker. The observation spaces may overlap and, in fact, may present data that, for all or most thresholds of the decision maker, would lead to different final decisions if considered separately. The likelihood ratios have been worked out for two such observation spaces, and have proven quite workable for comparing such situations as: (1) the final

* Primarily the work of Gordon Becker of Yale University, New Haven, Conn., and ITEK Corporation, Cambridge, Mass.

decision maker knows noise contamination of one of the subdecision makers is low, but is not sure which observation space goes with which subdecision maker; or (2) the final decision maker knows which subdecision maker goes with which observation space, but both subdecision makers are highly contaminated with noise.

Becker notes the flexibility in such a model and suggests, through the addition of feedback, a possible learning model. Immediate applications to group decision situations—such as conferences—are apparent.

The purpose of juxtaposing a psychophysical experiment and this conflicting information model is severalfold. First, may I emphasize the capability of decision theory as a useful analytic tool at several behavioral levels. This usefulness can be traced to its statistically-complete character, in the sense of Woodward,[4] and to the distinct separation between three sorts of quantities: (1) the physical or psychophysical probabilities, that is, those probability distributions that describe the physical stimuli; (2) the operational or a priori probabilities, that is, the probability that such-and-such a situation will occur; and (3) the costs or utilities, which usually represent numbers not derivable within the context of the situation itself, but rather weighting (stipulated or measured) given to various combinations of stimulus and response.

One of the most significant contributions of the decision-theory approach to date is the decisive reduction it is effecting in the arbitrariness of psychophysical criteria. In the purely physical and in the operational domains, especially when dealing with highly perturbed—that is, noisy—situations, reduction of criterion arbitrariness has really been essential to progress, and this may well be true in the psychophysical area.

Both of the studies discussed here were strongly system-motivated. In the psychophysical experiments, for example, the earlier success of decision theory in measuring processing instrumentation, applied both in the literature and through the use of Monte Carlo digital-computer techniques, generated the need for similar analysis of the psychophysical component in navy research systems. From the system point of view, an identical language for all processing components is absolutely essential, since system performance should be describable by the smallest possible set of numbers, preferably just one.

We see, then, a strong unifying potential in decision theory, linking human and nonhuman signal processing. A second area of unification is the capability for accurate description of varied states of knowledge of the decision maker. The description is accurate in the sense that the effect of a state of knowledge upon the decision maker's ability to make a correct (or least risky) decision may be accurately determined. In behavioral studies, such as the one mentioned on conflicting information, an accurate state-of-knowledge description becomes an all-important capability.

We may invert this last argument, however, and use the result to introduce the debit side of the ledger: that is, areas where decision theory is not completely satisfactory.

Accurate description of a state-of-knowledge is equivalent to the statement of a probability distribution (physical and/or psychophysical) over every space in a decision theory flow diagram; in addition, the so-called a priori probabil-

ities may have to be defined over a set of signal spaces. Statistically this means we have in decision theory a highly parametric tool the use of which demands either that we possess a rather complete statistical description of what is going on, or that we, in the absence of such knowledge, adopt some convention equivalent to the assumption of such a state-of-knowledge. Although it is possible to conduct a sensitivity analysis for each and every situation conceivable to us, such an approach only transfers the need for statistical description into the judgment of the analyzer, and so begs the question.

One asks, then, about quasi-parametric or even nonparametric decision theory. Since I am beginning to come dangerously close to the province of mathematical statistics here, I should like to restrict my comments to just one promising avenue. One way to categorize decision theory studies is whether they are concerned with fixed sample sizes or whether they involve sequential analysis; that is, a sequence of sample-process-decision-sample-process-decision, and so forth, until a final decision can be made. The criterion for number of samples is that the (maximal) error probabilities associated, on the average, with the final decision will be less than certain specified numbers. Usually, processor design is taken as an invariant from step to step. But if this invariance is not required and, instead, one permits the processor to learn its design as it measures the input, in the sense of Hofstetter's adaptive processors,[5] we have a technique that develops the statistics as it goes along. In fact, a useful transition from nonparametric to highly parametric theory may be possible in this manner. Clearly, many existing studies in learning theory and in perception are quite pertinent; a generalization of sequential decision theory combined with the results of these studies appears to provide a method of attacking basic state-of-knowledge difficulties in decision theory applications.

Closely connected with the state-of-knowledge difficulty is the practical restriction to low-order and normal—or, at least, relatively simple—statistics. For very practical and sometimes decisive technological reasons, we are interested in delimiting possible system performance. If decision theory is used, such delimiting is almost always sensitive to the statistics used. With normal statistics and the accompanying paraphernalia, such as low-order Markoff process assumptions, much has and can be done. Once we move away from these assumptions, however, decision theory as an analytic tool becomes quite cumbersome, and we may well question its usefulness, not on the grounds that it is a poor model of reality, but merely on the grounds that we cannot solve the numerical computations that the theory implies.

Intractable probability distribution functions are not the only source of computational difficulty. It is noteworthy that almost all the applications successfully carried through to date involve very low-order decisions: on the order of two to four. That is, in signal detectability we are concerned with making a second-order decision—one of two possible choices—or in the sequential detection problem, one of three possible choices. Since the number of likelihood ratios that must be computed grows roughly as the square of the order of decision, we may expect computational difficulties in high order decision theory, even for very straightforward statistics.

Both complex distribution functions and high-order decisions point toward a useful analysis that is dependent primarily upon statistical parameters instead

of upon the distribution functions themselves, and upon gross measures of the decision set: that is, toward a direct weakening of the power of the theory. However, the difficulty of high-order decisions can suggest a most interesting line of further theoretical development. For example, under fairly general assumptions, the optimum processor for an nth order decision may be described as a collection of $n(n - 1)/2$ likelihood ratio processors, followed by a large number of coincidence circuits. The coincidence circuits may be arranged in m levels, where $m = 2^n$. The output of this collection of likelihood ratio processors and coincidence circuits is the set of permissible decisions, and in general the outputs of a number of coincidence circuits will go to the same decision. It is an interesting property of this arrangement that every likelihood ratio processor and every coincidence circuit is essential in the sense that, for every point in observation space, one and only one decision is to exist. Omission of just one coincidence circuit, for example, means that at least one point will exist in observation space for which no decision is possible. Several properties of this matrix appear quite similar to those of existing models in perception; however, the point I should like to bring out is that, while computational requirements of high-order decision theory may force us to use gross or average measures, this difficulty—as with the other difficulties—is often heuristic. It may, as in the case cited above, not only lead to unexpected and possibly useful gross measures, but it may also provide insight into techniques that have developed from wholly different points of view.

Summary

Having gone fairly far afield in this last discussion, I should like to stop and summarize the point of view developed in this paper regarding the application of decision theory.

We have inspected, briefly, a psychophysical experiment and a behavioral model to bring out the flexibility and unifying potential of decision theory. We have seen that through clear separation of three types of measures—the psychophysical probabilities, the a priori or operational probability, and the cost—criterion arbitrariness may be reduced, but human observers do not always follow one-to-one with the dictates of the theory. Continued successful application of decision theory in nonhuman components of research systems generates a need for a similar approach—or at least, a similar language—for human signal-processing performance. On the debit side, there are difficulties due to the parametric nature of decision theory, such as state-of-knowledge descriptions, and computational difficulties that arise in the use of other-than-normal statistics and in high-order decision theory. While these difficulties exist, in many cases promising directions of further research, such as adaptive processing from sequential decision-theory analysis, are indicated. In my opinion, we are at present only beginning to realize the usefulness of decision theory in systems applications involving human decision making.

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SOME PROBLEMS IN VISION REGARDED WITH RESPECT TO LINEARITY AND FREQUENCY RESPONSE*

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Introduction

In general there is a considerable difference between the methods used in psychophysical research on vision and those of electrophysiology. The work done on threshold vision can be considered the main contribution to our knowledge of the visual system, although recent work on more complicated tasks, with respect to afterimages, also seems to gain more importance. The study of absolute as well as incremental thresholds leads naturally to the use of small signals, culminating in the important work of De Lange on critical flicker fusion (CFF) with modulated light. Electrophysiology, however, has been studied chiefly with large signals, especially flashes, even when background illumination (light adaptation) was used. This gives rise to large and highly synchronized nervous signals.

The present study describes the use of comparable signals for psychophysical and electrophysiological measurements. My associates and I call this the weak signal study.

Although our own electrophysiological work deals only with slow potentials and is still far from complete, the signal transmission could be followed with regard to linearity; it has been proved that the method of linear approximation introduced by H. De Lange (1957, 1958) permits better comparison between subjective and objective measurements in man. A special feature is that, under favorable conditions, responses can be recorded that are below the subjective threshold. A simple discussion of the statistics of threshold flicker will support the hypothesis that for small incremental signals an approximately linear character is maintained to a later stage in the process.

This kind of treatment appears promising, but it has been shown, on the other hand, that the decision problem even in the simple process of flicker fusion is complicated and far from solved.

Flicker Fusion

Until 1955 the data on flicker fusion and flicker responses in the electroretinogram (ERG) and electroencephalogram (EEG) were not unified by any inclusive theory. De Lange (1957, 1958) introduced the use of modulated light and gave us a definite analysis of this problem. By this modulation it is possible to change the stimulus (depth or frequency of modulation) without changing the adaptation of the retina. A plot of frequency versus depth of modulation for flicker threshold at different intensities can be made.

FIGURE 1 reproduces one set of De Lange's psychophysical curves, which were measured with a field of 2° with an equal surrounding for threshold vision

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at different intensities. These curves show an extremely high attenuation of frequencies above 15 cps. De Lange argued that at these threshold conditions the visual system behaves linearly with respect to the stimulus, and he proved also that at these frequencies the threshold is determined only by the ampli-

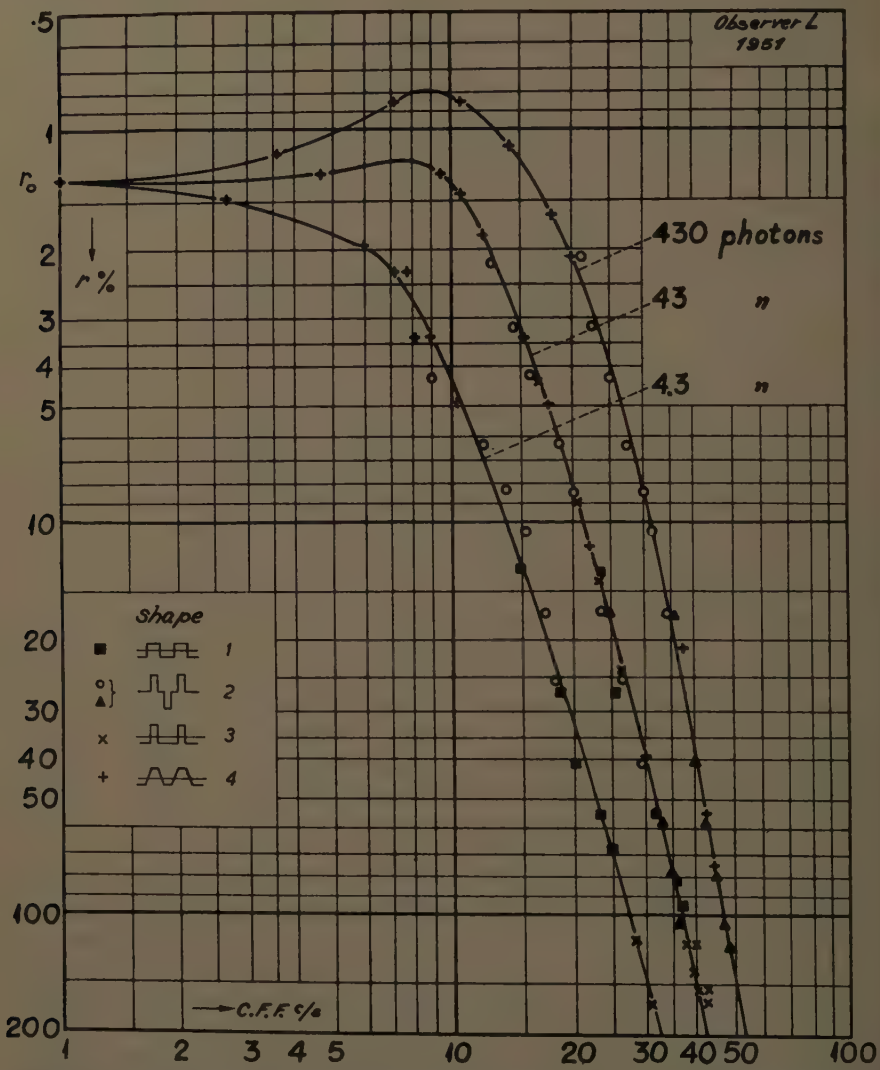

FIGURE 1. Attenuation characteristics for white light of the right-hand system fovea brightness perception: 2° test field, with a large surrounding field having the same luminance as the test field, and central fixation. There are four variations in the luminance as shown: 1 and 4 are the shape of modulation, and 2 and 3 are the mode of interruption. No. 4 is approximately sinusoidal; ripple-ratio r = fundamental B_1/mean luminance B_a of the stimulus; CFF = critical flicker frequency; r_0 = the critical value of the variation at the site of the threshold mechanism, measured at the input if no attenuation in the system occurs; the attenuation $\eta = r_0/r$. Reproduced by permission from De Lange, 1957.

tude of the first Fourier component. This is true only if the harmonic content is not too high (Levinson, 1960). High frequency attenuation must occur before the linear transformation is disturbed. From Veringa's work (1958) it can be concluded that this linear character is approximated, even at modulation percentages several times above threshold. We shall come back to this linearity problem later.

Successive and Simultaneous Contrast

Most of our own measurements have been performed by a modulated television projection tube as described by Denier van der Gon *et al.* (1958). The maximal luminous fluxes (sometimes used in electrophysiological work) were about 1 lumen in a field of 1 steradian. They were produced by projecting the light source through a condenser into the pupil of the eye (Maxwellian view).

In direct view, an annulus is presented of 8° inner diameter and 2° width, representing an apparent brilliancy of maximum 200,000 lux equivalent. For lower intensities, filters were used and usually the tube was run at a lower level. We did not enter scotopic conditions. The light output could be modulated according to any electrical signal up to frequencies of 100 cps, and flashing techniques with regulated average intensity also could be used.

The psychophysical experiments were usually performed by presenting the light from the modulated source through a hole in a white screen of 20 × 20 cm. Surrounding fields were generally made by conventional projectors with adjusting blue filters to compensate for the difference of color. Usually the subject was at a distance of 3.5 m. and we used a pinhole pupil of 1.5 mm. The thresholds were determined from below and from above flicker vision. Under most conditions this determination could be made with great precision; in small fields without surround, at low intensities and at low frequencies, a determination takes longer, and the accuracy is less. The curves presented give an impression of the accuracy of our results that we consider good enough with respect to the conclusions and quantitative relations derived from them. Therefore we preferred fast determinations of a whole family of curves to a more accurate approach for each one separately. Most of this work has been described in detail by Denier van der Gon (1959).

In FIGURE 2 a rise of threshold for large fields is shown at A, with a black surround below 10 cps. FIGURES 3 and 4 complete these findings also for smaller fields and show the relation between the thresholds with and without surrounding field and for different visual angles. Kelly (1959) presented curves in which the flickering field faded out toward the periphery, and he reported an occasional very steep fall below 15 cps. However, we could not reproduce his results, perhaps because our experimental conditions were not the same.

For large fields (>2°) the main difference between the results with a black surround or an equally illuminated field is found in the frequencies below 10 cps. We assume that in the case of equal surrounds, at low frequencies, a real spatial contrast determination is made by the subject. For small fields, thresholds are usually higher with a black surround than with an equal surround. In this case also frequencies >10 cps are influenced.

FIGURE 5 shows the dependency of the threshold of 12 cps flicker on the illumination of the surrounding field. A maximum sensitivity (minimum threshold) is found when there is no difference between surrounding and flicker field. However, when a black ring is introduced between the two, a minimum can still be found, although it is not so low. The surrounding illumination, however, has to be increased manifold to reach such a minimum. In this case there is nowhere else a field that can be used for comparison. Therefore, in our opinion, the lowering of the threshold here is a result of the total illumination

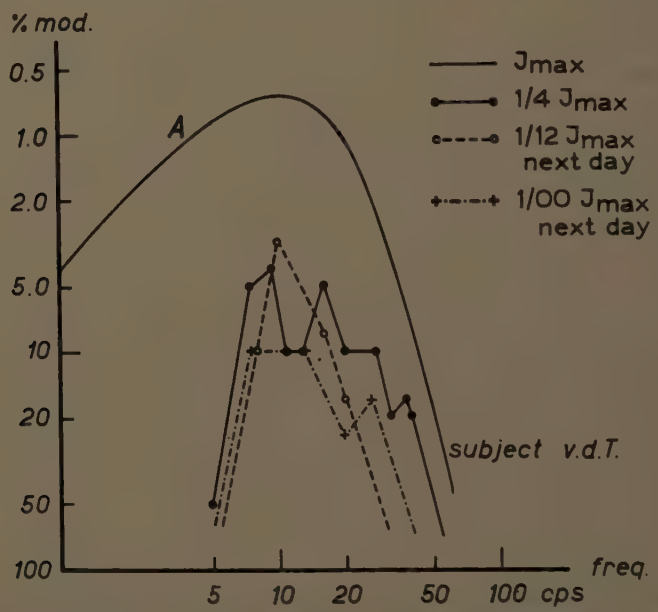

FIGURE 2. Threshold modulation depth as a function of frequency (De Lange curves) with subjective determination, on the basis of electrophysiological responses with depth-electrodes. Although the electrophysiological curves have a higher modulation percentage, which can be ascribed to the disturbing influence of the spontaneous activity, the high-frequency portion is comparable to that in subjective vision whereas the low-frequency portion is totally different. Reproduced by permission from *Acta Physiologica Scandinavica* (Kamp *et al.*, 1960).

condition of the retina, and contrast determinations at frequencies >10 cps are chiefly successive contrast determinations. The influence on the sensitivity is a function of the distance. It is improbable that these effects are primarily results of light diffusion in the eye. Within this concept it is also understandable why fields of $>2°$ at 10 cps and higher do not show much difference, whether a black or a white surround is used. The influence of the surround in the case of small fields is here probably replaced by the influence of the illumination of the field on itself; this field is large compared with the distance at which this kind of influence becomes negligible.

The dependency of the differential successive threshold on the surrounding field was also found for single flashes (positive and negative).

Our explanation differs considerably from that of Kelly (1959).

We may conclude that at frequencies greater than 10 cps all flicker measurements are successive threshold determinations, the highest differential sensitivity occurring with an evenly illuminated retina. At and near threshold there are no large deviations from linear functioning. A reduction of high frequencies occurs at an early stage.

Simple Quantitative Considerations

We shall see that it is possible to approach the flicker fusion problem electrophysiologically and even to find responses below subjective threshold. Although, owing to lack of knowledge about such fundamental data as quantal efficiency and integration time, a discussion can be only general, it still seems useful to reconsider the CFF threshold concepts and functioning in a semiquantitative way.

Since Rose (1948, 1953, 1957) and De Vries (1943), there has been much discussion about the optimality of the visual system with respect to physical limitations caused by the quantum character of the light. Most experiments have been performed on incremental or absolute threshold vision and, although often contradictory in details and interpretation, they indicate that with reasonable assumptions of quantum efficiency and summation times the functioning at moderate intensities cannot suffer too much loss of information. Denier van der Gon (1959) studied the problem of visual acuity with artificial pupils, enabling a mathematical approach to the quantum distribution at the retina. He could describe the experimental results with the probability that in one summation time more quanta were falling in a light detail of, for example, a Landolt C, than in a dark detail. In this way his results could be translated into the statistics of incremental thresholds. To give a macroscopic impression of detectability as a function of the statistical probability, FIGURES 6*a* and *b* represent a Landolt C with its gap to the left, which has been constructed by random arrangement of the dots. The light quanta are shown as black dots; thus the figures can be compared directly with a Landolt C. On the average, three quanta fall in a light part and one in a black detail in one summation time. The probability that more quanta will be found in a light detail than in a dark one is 78 per cent in this case. This probability, given by the distribution function of the difference between two Poisson variates, can be estimated with help of a normal distribution and may be written as a function of the ratio k between the mean μ and the square root σ of the variance of this normal distribution: $k = (n_1 - n_2)/\sqrt{(n_1 + n_2)}$. In the given case ($n_1 = 3, n_2 = 1$), an approximation can be found by $\mu = 2$ and $\sigma = 2$. If $\Delta n = n_1 - n_2 \ll n$ and $n = (n_1 + n_2)/2$, this formula yields $k \sim \Delta n/\sqrt{2n}$. From the set as presented it is nearly impossible to tell where the gap would be. The figure composed of the 10 images, described by $k = 3$ (probability >99.8), shows that if no data about the possible positions and shape of the gap are given a high probability to exceed is needed to ascribe a pattern to such a distribution. We are quite aware that this representation might have a primarily didactic value.

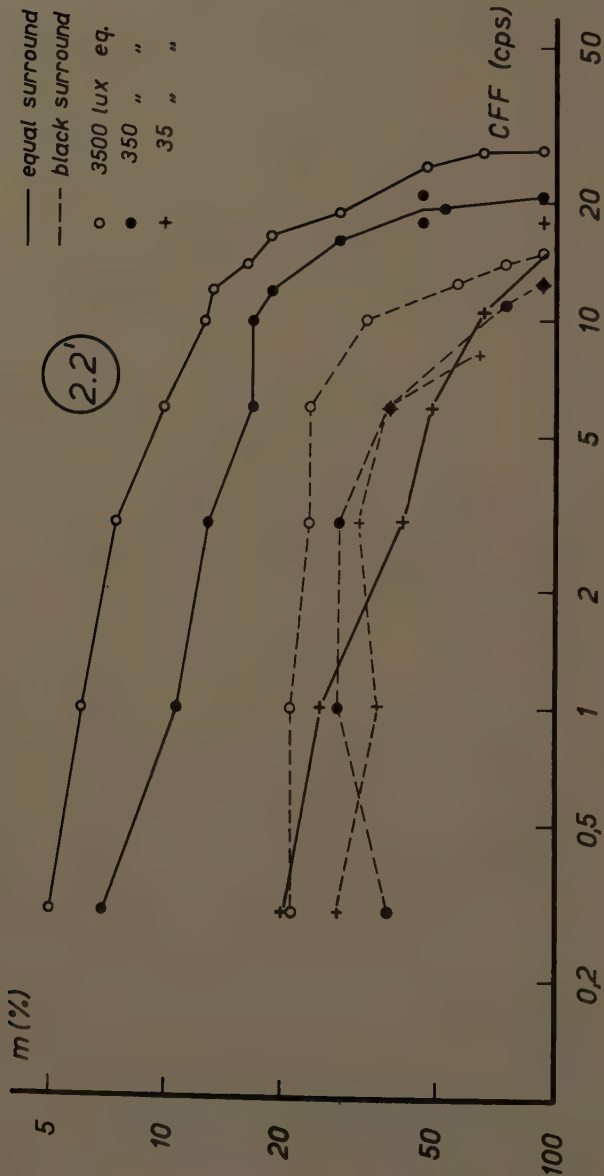

FIGURE 3. The plotted curves give the relation between the frequency and depth of modulation at different intensities in two visual fields. An artificial pupil, 1.5 mm., is used. The general trend is that for smaller fields the depth of modulation for the CFF is higher at the same frequency and intensity than for larger fields. This is generally true also for thresholds measured with an equal surround. The largest differences are found for large fields at low frequencies. Adapted from Denier van der Gon, 1959.

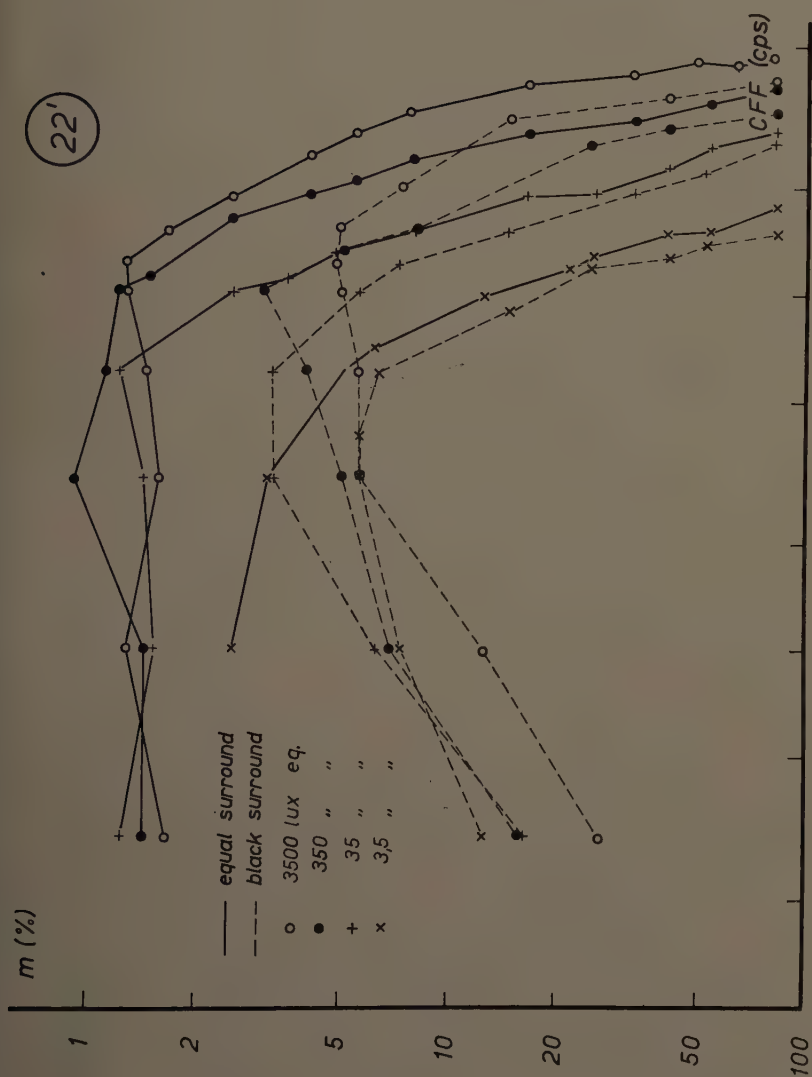

FIGURE 4. See legend for FIGURE 3.

We now reconsider FIGURE 4. At 10 cps (350 lux equivalent, pupil 1.5 mm., retinal illumination 200 trolands) the threshold is found at 1 per cent of modulation for a field of 22′. As we have said, under these conditions (equal surrounding field) the flicker problem may be treated as a purely successive phenomenon. Contrary to most situations in incremental threshold studies, the surfaces to be compared may be assumed to be equal in this case. A simple assumption would be that for CFF the criterion will be the probability that the positive part of the sine wave contains more quanta than the dark (negative) part. Since at 10 cps the system seems to achieve its highest sensitivity,

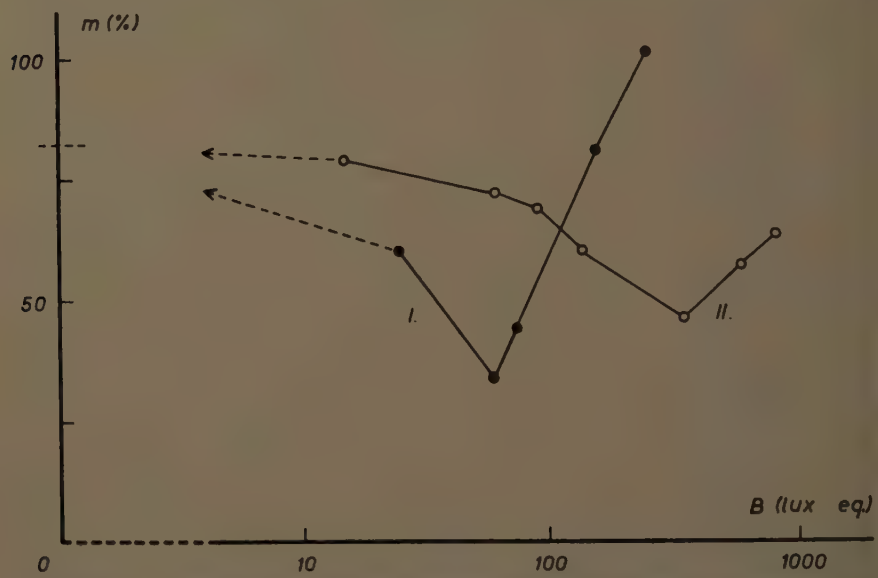

FIGURE 5. Critical modulation depth dependent on the surround: the frequency is 12 cps; the brilliancy of the test field is 60 lux eq.; and the artificial pupil, 1.5 mm., is used. Curve I represents the surround without interruption and Curve II is the surround interrupted by a black annulus of 24 min. The sensitivity (reverse of the modulation percentage for the threshold) reaches a maximum in both cases but, when a black annulus is introduced, shifts to a much higher value of illumination of the surround. Adapted from Denier van der Gon, 1959.

a summation time of 50 msec. is chosen, differing not much from the conventional assumptions at these illumination levels. With an over-all (foveal) quantum efficiency of 1 per cent for the white light used (5500° eq.), the average number of quanta effectively absorbed will be $N = 4.2 \times 10^4$ quanta/50 msec. The net difference between two sine-halves at 1 per cent modulation depth for threshold will then be $D = 5.4 \times 10^2$ quanta $= 1.85 \sqrt{2N}$, which yields a probability of finding more quanta in a positive than in a negative sine-half of ~97 per cent (see FIGURE 6*b*). I emphasize at this point that the eye probably has no special constructions for detecting temporal periodicity and that the problem therefore can be treated on the chosen statistical basis. The highest estimate for quantum efficiency for white light might amount to 4 per cent. We chose a conservative value, however. A higher efficiency would not change

essentially our discussion, which mainly serves to determine whether optimality considerations might also be useful here. Considering also the possibility of the influence of a memory, the result is a reasonably safe device for detection.

We do not think it fruitful to present a model of the way in which the temporal comparison is done. An interesting phenomenon, possibly connected with this process, is that subjective discrimination of the frequency of flicker is possible only up to about 18 cps, if adjusted in the depth of modulation with

FIGURE 6. Quantum distribution at the retina for Landolt C (reversed). In a light detail (equal in size with the gap), an average of three quanta are distributed in $\frac{1}{15}$ sec., compared with one in a dark detail. Ten exposures of $\frac{1}{15}$ sec. each are presented in (*a*). The probability of finding more quanta in a light detail than in a dark one is 78 per cent. Even when the whole set is presented, it is extremely difficult to find the direction of the gap. It is possible, but still difficult, to find the gap (directed to the left) in the figure (*b*) composed of all 10 presentations. Here the probability to exceed is 99.8 per cent. If this kind of presentation is allowed for comparison with threshold vision, it shows, as Rose did for contrast, that a high probability to exceed is needed to perform a judgment. Adapted from Denier van der Gon, 1959.

regard to the CFF curves. At higher frequencies the discrimination fails; the low subjective frequency of flickering fluorescent tubes is well known. It is tempting, therefore, to assume that the summation differs from the normal integration caused by the high frequency attenuation in early parts.

"On" and "Off" Concepts

We shall restrict this discussion to the threshold problem, since the importance of "on" and "off" elements in man for important biological functions, although not explicitly shown, could easily be accepted.

The importance of these elements for visible acuity and contrast phenomena is sometimes explicitly stated (Ditchburn, 1959; Kelly, 1959), and sometimes implicitly, but it is not quantitatively argued (Le Grand, 1956). The generalized idea is that by movement of the eye, spatial gradients are transduced in temporal events. Therefore a comparison with flicker, where all events are temporal, is interesting. Doubt about it can be found in the work of Denier van der Gon (1959) and, very recently, in that of Hebbard and Marg (1960). The importance of the micronystagmus is sometimes argued, considering the results obtained with stopped images, in which case the visual impression tends to disappear. The disappearance time of the image, however, is far greater than the time needed for reaching a high visual acuity. Moreover, it is obvious that, in an optical system, moving of the image impairs detectability except when information about the position is available. This is the case only for larger saccadic movements. That the latter have a function for the maintenance of vision, however, seems very reasonable.*

The stress usually laid on the on and off elements is a natural consequence of the common experimental technique in electrophysiology, using large and sudden signals. In the case of modulated flicker, an influence of on and off elements must be considered in a generalized way, since we deal only with more or less slowly varying stimuli. In fact, every successive contrast decision is an on or off decision. This applies equally to flicker and to single flashes. It is not easy to extract from the literature what times are involved in establishing an on or off effect, most results having been gathered with rather large transients. To make a sensible distinction with the usually accepted general summation time of the visual system of 50 msec. we have chosen a "differentiation" time of 10 msec. This is probably not too long, compared with the micronystagmus frequencies presented in the literature, which are quoted up to 200 cps (when using conventional calculus, 10 msec. would be equivalent to $100/2\pi = 16$ cps). The successive judgment described earlier with a summation time of 50 msec. was performed according to a difference of 5.4×10^2 quanta. If the comparison must be done each 10 msec., according to our assumption of on and off times, the statistics will be impaired by about $\sqrt{5}$ at a frequency of 10 cps. We see that from $k \sim 1.8$, we reach $k \sim 0.8$, which means a heavy loss of discriminability. Even when a higher quantum efficiency had been assumed (compare with FIGURE 6), this would be important. (If this simple model for successive comparison could be used, it is easily seen

* After this manuscript was completed my attention was directed to a paper by Riggs (1958), in which a very clear review of this problem can be found.

that at low and at high frequencies attenuation would occur. At high frequencies, however, "resonance" phenomena would be expected, but could not be found in our experiments with white light.)

From this we may conclude that psychophysical and elementary quantum considerations, to be supported by our electrophysiological results, make it plausible that the flicker threshold judgment is done at a rather late stage (Rushton, 1953; Brindley, 1960). After the decision has been made, the linear relation between signal strength (modulation depth) and response is taken over by a nonlinear relation. An early place for this transformation is not easily brought into accordance with our finding of subthreshold responses in cortical derivations and our other results. In our simplified theory on and off effects would certainly not improve threshold vision. Their use might be found in editing important and larger signals in a safe and economical way.

Logarithmic System

Generally the visual system is considered to exhibit logarithmic properties. Of course this logarithmization is only an approximation, as are nearly all mathematical formulations of physiological phenomena. Certainly in the nonlinear transformation two features may be distinguished. We call these the immediate logarithmization and the slower adaptational process. Data on the pupillary reflex of man, that correct earlier material, show also that for initial steps, over a large range, an approximate logarithmic functioning occurs. FIGURE 7 shows the pupil contraction as a function of the intensity of the light step. As may be expected with logarithmic functioning, at low intensities deviations must occur and, at high intensities, some extra saturation is present.

After a longer time the light adaptation diminishes the sensitivity of the (primary?) system still further, without too great an increase of the statistical fluctuations, however.

Furthermore, because of the use of suprathreshold modulations in our electrophysiological experiments, we shall discuss some single properties of logarithmic systems, abstracting from actual processes.

The finding of a more or less linear transmission of increments can be brought into reasonable accordance with logarithmic function. The development of $\log I\ (1 + M\cos \omega t)$ yields $R = \log I + M\cos \omega t - (M^2/2) \cos^2 \omega t + (M^3/3) \cos^3 \omega t - \cdots$. I is a relative measure of the intensity and, in our case, is supposed to be much larger than 1. For modulation depths of 25 per cent the amplitude of the second harmonic ($M^2/4 \cos 2\,\omega t$) is only $1/16$ of the amplitude of the fundamental in the response, and the average shifts from $\log I$ to $\log I - M^2/4 = \log I - 0.015$.

$$M^2/2\ \overline{\cos^2 \omega t} = M^2/2\ (1 - \overline{\cos 2\,\omega t})/2 = M^2/4.$$

Furthermore, a property of a logarithmic system is that the response to a constant depth of modulation is independent of the intensity. The depth of modulation of the response will be, in our approximation, the $M/\log I$ for small modulations. Nevertheless, at high intensities, where the depth of modulation of the response is smaller, the subjective discrimination is better; the approximate modulation threshold in a considerable range is $M \sim 1/\sqrt{I}$. It seems

also that the threshold setting is done by the quantum fluctuation itself or by a kind of learning, by which the visual system has organized itself not far from optimal (Brindley, *op. cit.*, p. 196, discusses this question).

The problem of linearity as discussed by De Lange with respect to Talbot's law can also be treated in the same way. Talbot's law can be expressed in the simplest way by the following. Light seen as stationary has the same intensity as the physical average. Thus in FIGURE 4 we see that at 50 cps the CFF will

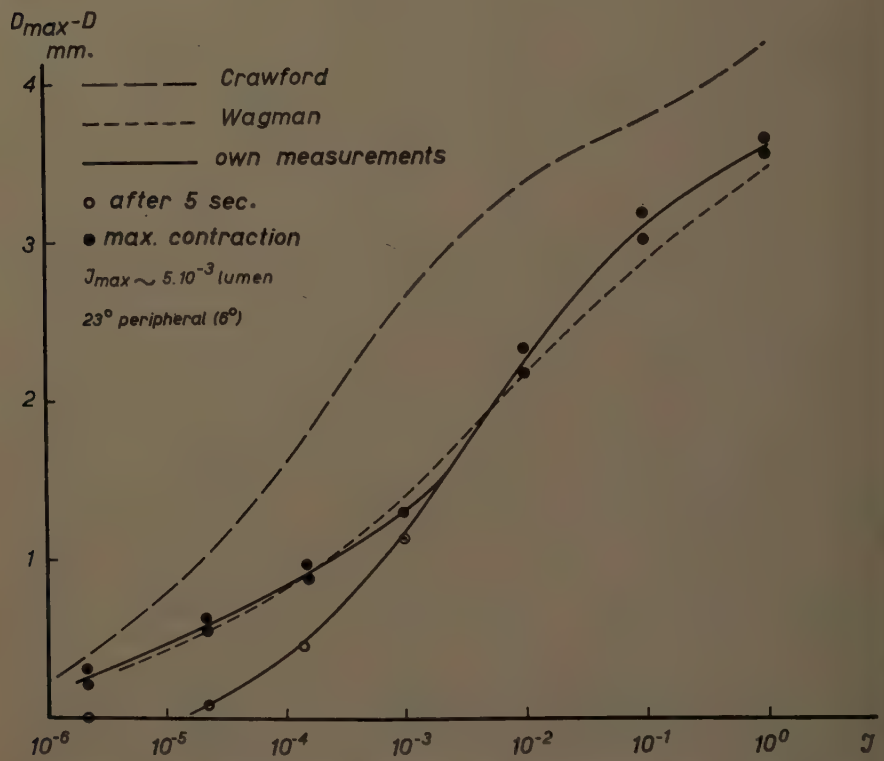

FIGURE 7. Pupil contraction as a function of flux, compared with data of other authors. In the present material there is a clear distinction between curves for maximum contraction and for contraction measured after a certain time (rebound). Reproduced by permission from *Acta Physiologica et Pharmacologica Neerlandica* (Van der Tweel and Denier van der Gon, 1959).

be at 200 per cent modulation (short flashes). The threshold at very low frequencies is here 1.5 per cent, which may be considered a real simultaneous threshold at these low frequencies. Log 1.015 $I \sim \log I + 0.015$. This is the same difference as that found for the shift of the average for a depth of modulation of 25 per cent. From our simple formula we see now that if at 50 cps an eightfold attenuation occurs, the depth of modulation of 25 per cent remaining from the original flashes would result in a shift of the average level equivalent to a 1.5 per cent shift of the original intensity, also just at the level of detectability. If Talbot's law is not discarded yet, this proves that there

is enough high-frequency attenuation before nonlinear processes play a role, but this attenuation need not be more than eightfold at 50 cps while, subjectively, according to our curves, the sensitivity has decreased about 150 times.

Electrophysiological Experiments

With different methods of analysis and of recording, electrical responses to modulated light and sudden changes in illumination have been studied (Van der Tweel *et al.*, 1958; Kamp *et al.*, 1960; Van der Tweel and Visser, 1959; Visser *et al.*, 1960). Electroretinograms (ERGs) were recorded in cat and man with simple averaging and also with sensitive integration (integrator of A. Rémond). As many as 2000 responses have been used for integration at high frequencies. Occasionally experiments were done on the lateral geniculate body of the cat and with intracortical electrodes in man, all with selective amplification. A number of integrated curves have been recorded with normal scalp electrodes. Some of our results most relevant to the present discussion will be reported.

The fields used were always rather large, not only to get large responses but also to decrease eventual influences of stray light. The retinal illumination used in man was not exactly known. In addition to other instrumental problems the contact glasses made it difficult to obtain an even illumination of the retina. The losses in the contact glasses were not always the same, and the natural pupil had to be used because a continuous use of mydriasis produced too great a strain. This may account for a certain variability in the response curves. No differences were found, whether the room was moderately illuminated or not. Certainly improvement of the optics would be useful and, with the development of new techniques, it may become possible to measure responses with smaller fields in conditions comparable to foveal flicker experiments. The subjective curves measured under identical conditions, as in our electrophysiological experiments, however, show no important deviations from those obtained in smaller fields. Only in two cases was mydriasis used to see whether phase shifts found at low frequencies could be attributed to the pupillary reflex. We found no difference that could not be attributed to differences in luminous flux. When the nonrecorded eye was stimulated no "responses" were found even with a great number of integrations.

The retinal responses with fields of 1 steradian and illumination levels of about 200 to 100,000 trolands were approximately sinusoidal in cat (FIGURE 8) and in man at depths of modulation of 25 per cent or less. Although responses of less than 10 μv are easily disturbed, especially at low frequencies, by noise, the amplitude and phase characteristics were determined with reasonable accuracy. They are shown for one human subject in FIGURES 9*a* and *b*. Some facts seem important. To a first approximation, the ERG behaves linearly at these illumination levels and with modulation percentages up to 25 per cent. This is also proved by the recording of responses to positive and negative step functions. These show a fair correspondence with the responses calculated from the phase and amplitude characteristics (FIGURE 10*a* and *b*) despite the irritability of the eye in these long sessions. At 25 per cent modulation there are, however, in addition, nonlinearities, as shown by FIGURE 11, in which responses to positive and negative incremental steps are shown. At 12.5 per cent this seems to have improved, but we could not determine whether all

nonlinearities would disappear; with smaller steps the noise became relatively too large. The time constant of 0.7 sec. did not permit us to follow the step response longer than about 300 msec. Recent measurements by Schweitzer and Troelstra in the Institute for Perception, Soesterberg,* with DC recording make it probable that the predicted long-lasting zero shift indeed occurs, al-

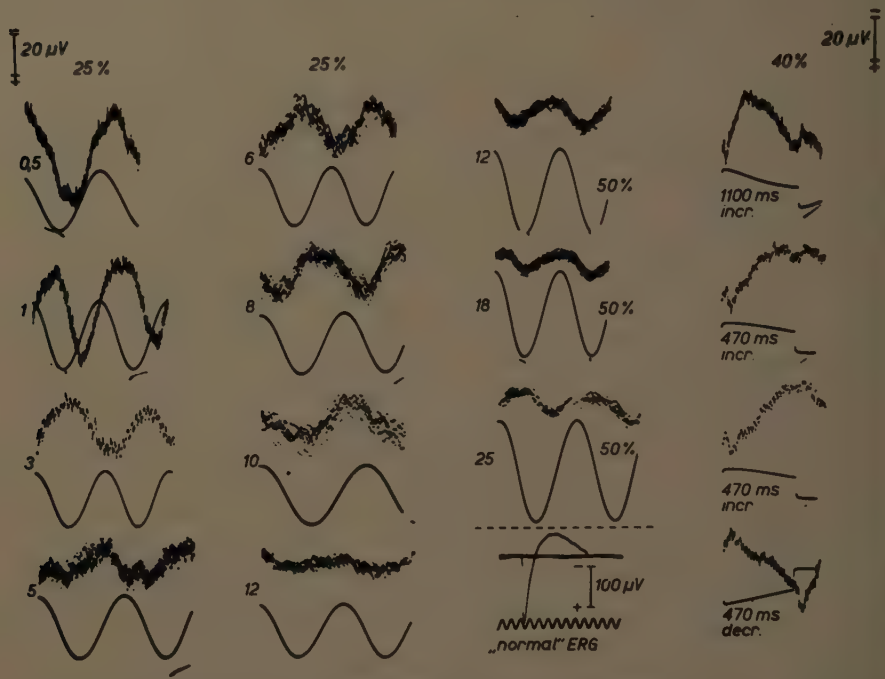

FIGURE 8. A set of frequency responses in the electroretinogram (ERG) of the anesthetized cat (pentobarbital): the retinal illumination was about 50,000 trolands and the field ~56°. The upper trace of each pair represents the ERG higher frequencies with superposition. The lower trace represents the AC output of the photocell controlling the light source. The phase shift at 3 cps is already 180°. At 5 cps, the amplitude is lower than at 3 cps and, at 6 cps, a distortion is clearly visible. At 12 cps the modulation had to be increased. The "normal" ERG is recorded with a step from dark to light. The last vertical row shows incremental step functions. The responses to positive and negative increments are reasonably symmetrical except perhaps for the first rapid deflection. Reproduced by permission from the *Proceedings of the International Symposium on Electroretinography* (Van der Tweel and Visser, 1959).

though here also our data do not permit a definite conclusion about the symmetry for positive and negative increments.

It is difficult to choose certain features from the phase or amplitude characteristics or, even better, from the $R \cos \varphi$ curves, that could correspond to physiologically distinct processes (these curves, constructed from the amplitude R and the phase φ, were used to calculate the step response). The same applies to the conventional way of distinguishing characteristic waves in step or

* Personal communication.

impulse responses, which are also totally descriptive for the system in the linear case. It is not clear which combination of functions is incorporated in the appreciable phase shift at low frequencies. It is tempting to distinguish at least a slow and a fast process, perhaps in the conventional way, since the phase and amplitude characteristics cannot be described by a minimum phase shift network or by a simple delay.

The amplitude relation between low and high frequencies changes with the intensity. Normally the higher frequencies at high intensities are relatively enhanced. At a frequency of about 10 cps the response is nearly intensity-independent and, on the whole, the intensity is not found to be an important factor in the amplitude of the response below 15 cps. This supports the conventional assumption of a more or less logarithmic function. At high intensities, the attenuation of high frequencies is much less than in subjective vision. The results in these frequency regions show a high reproducibility.

It may be significant that at higher frequencies subthreshold retinal responses could be recorded. With 2000 integrations, a response could be recorded at 80 cps and 25 per cent depth of modulation, as shown in FIGURE 12. With so many integrations one always must be careful with respect to eventual photoelectric effects. The dependency of the phase on the frequency makes it improbable, however, that the response was not due to retinal activity. In a number of experiments that we succeeded in recording at low depths of modulation but above threshold, the responses were still approximately proportional to the modulation. It will be most interesting in these cases to see whether distinct alterations will appear on passing the subjective threshold. With regard to this it is important to remember that the fields used were large compared to the integrative properties for subjective vision.

Lateral Geniculate Body of Curarized Cat

These experiments were meant only as preliminary ones to see whether modulated light could also be used in these regions of the optical pathways. Electrodes were brought into the lateral geniculate body (l.g.b.) of curarized cat (operated under local anesthesia) by the Horsley-Clarke technique. For most of this work a high illumination of about 50,000 trolands and a field of 15° was used. We present some of the results here because they indicate that in the l.g.b. a certain linearity is maintained, and we learned something about harmonic distortion.

There was a great variety of responses but, at a number of places, using selective amplification ($Q = 20$), response curves were measured. The accuracy of the curves is not very high because of the inherent noise that, however, when measured with an averaging rectifier, was not too dependent on frequency. FIGURE 13 gives some of our results. It is seen that there is attenuation at the low and at the high-frequency side. Modulations down to 0.5 per cent could be traced in the response. A high second harmonic content was found, much higher than could be accounted for by ascribing the distortion to a logarithmic function. The harmonic content was increased at higher modulation percentages. At some places a linear relation with the modulation was found up to a certain percentage. In one case, at 22 cps, with modulation of more

than 20 per cent, the responses at the proper frequency disappeared, but at the same time 44 cps was found with a large amplitude. The response at these frequencies in this well-controlled experiment is presented in FIGURE 14. At some places below 5 cps no slow responses were found, but in the loudspeaker deep modulations of the light produced a hissing rhythmic sound (frequency-

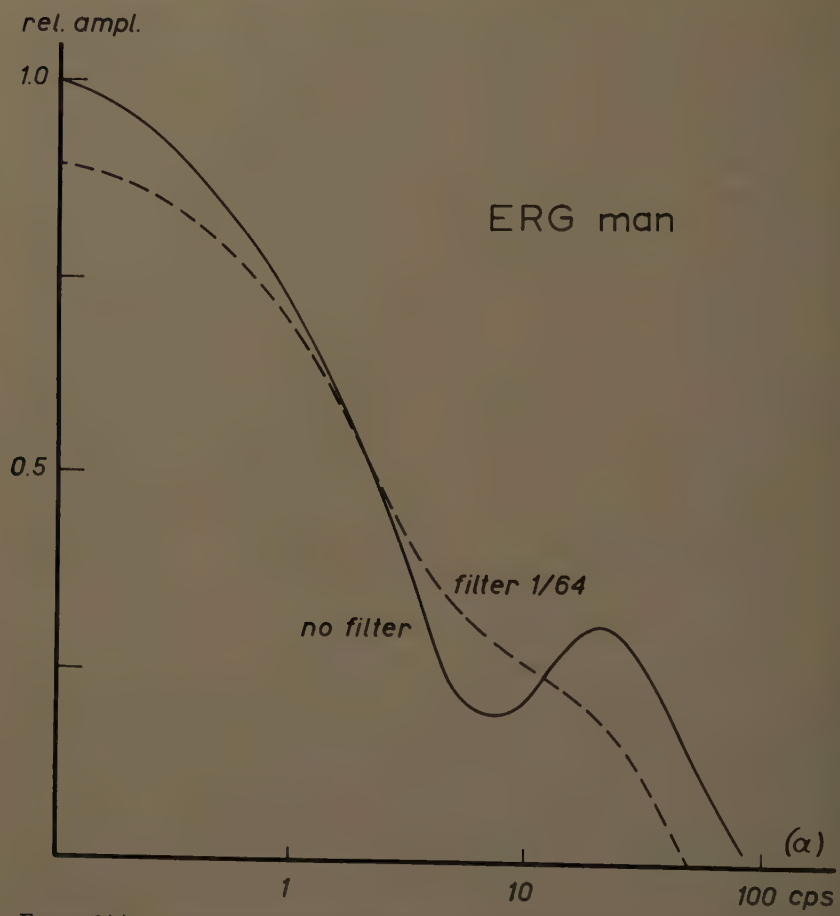

FIGURE 9(*a*). Amplitude (*a*, *above*) and phase response (*b*, *opposite*) in the electroretinogram of man, recorded with integrating technique: visual field ~56°; natural pupil; and maximum brilliancy ~20,000 lux eq. Note the relatively low attenuation at high frequencies compared with FIGURES 3 and 4. The amplitudes do not differ substantially for the two intensities. The amplitude accuracy below 3 cps is not very high. The considerable phase shift at low frequencies becomes still higher at low intensities. On the amplitude scale 1.0 is equivalent to about 10 μv.

modulated action potentials). Apparently there were at least two representations of the modulated signal. From a recent experiment with lower intensities (about 200 lux, diffuse on the eye) in several points of the optical radiation and l.g.b., confirmation of our previous results was obtained. FIGURE 15

shows responses at different modulations for the fundamental and the second harmonic. At 10 per cent the fundamental is larger than at 50 per cent, while the latter evokes greater second harmonics. The important results show that large signals are not always the best means of evoking large potentials and that complicated interactions take place when the stimulus exceeds certain levels. Of course these results do not suggest that this is the only way in which the system operates.

Objective Recordings in Man

This part of the work has been done in collaboration with several institutes. A patient of Sem-Jacobsen's who had some of the electrodes implanted in an

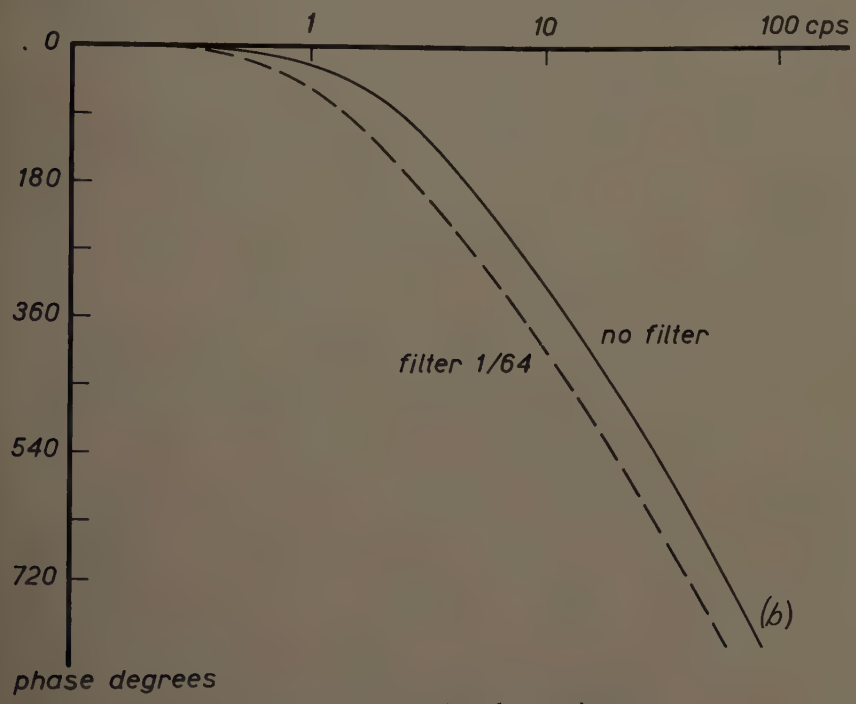

FIGURE 9(*b*). See legend on the opposite page.

optically sensitive primary part of the cortex was the subject. The patient was considered for leucotomy, and the depth electrode technique was used to restrict the area to be excised (Sem-Jacobsen *et al.*, 1956). We used a modulated television tube apparatus. The light was projected in the conventional way onto the eyes of the patient. The natural pupil was used. Normally a range of 500 to 50,000 lux eq. was used as a brilliancy of the presented annulus. Although we had to use the technique of selective amplification and, at this stage of our research, still tried to measure "De Lange" curves with reference to the noise, we present these results also because we think it was rather important that, with modulated light, responses have been recorded intracortically

in a fully conscious man. The detailed technique and results have been described elsewhere, but some of these results bear on the questions treated in this survey.

FIGURE 16 shows the selectively amplified signals of three different intra-

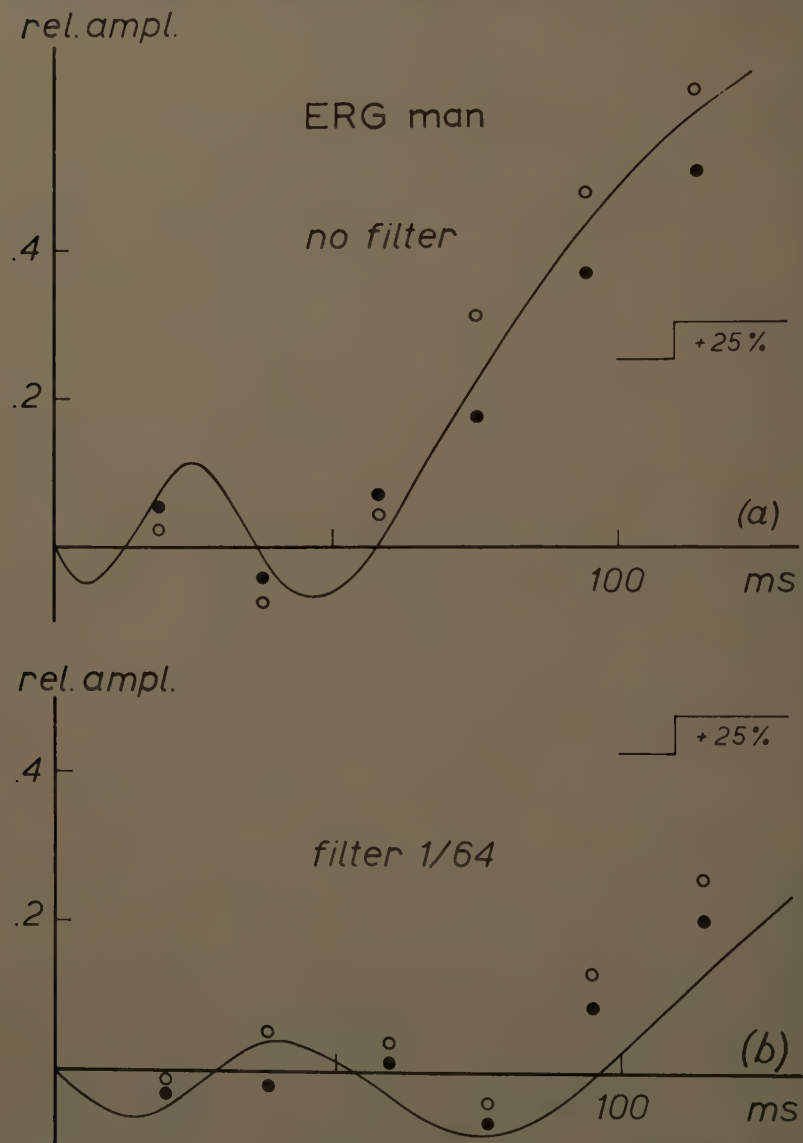

FIGURE 10(*a*) and (*b*). Calculated step responses from the data in FIGURES 9*a* and *b*. The points are recorded from two runs of 50 integrations at 25 per cent increments. They are adjusted in amplitude but not in time. The different "waves" are in good correspondence in the theoretical and recorded curves. The higher phase shift at lower intensities is also clearly reproduced in slower curves.

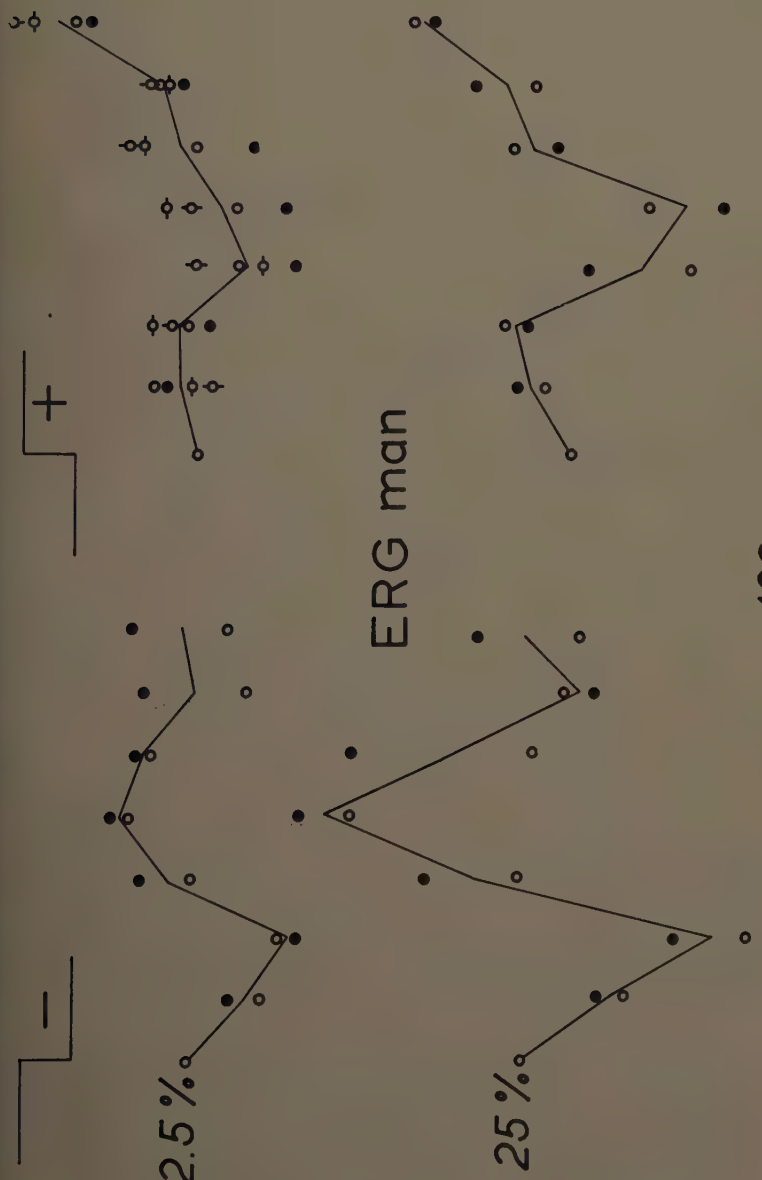

FIGURE 11. Recorded step responses with shorter intervals between measuring points. The negative and positive responses are different in size but reasonably alike in shape. Whether a better symmetry will be reached at still lower depths of modulation cannot be determined from the material.

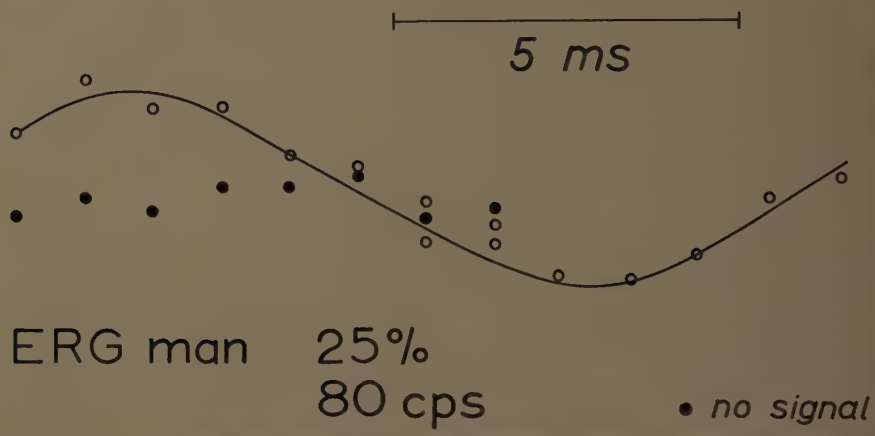

FIGURE 12. Subthreshold response at high frequency under the same conditions as those in FIGURES 10 and 11.

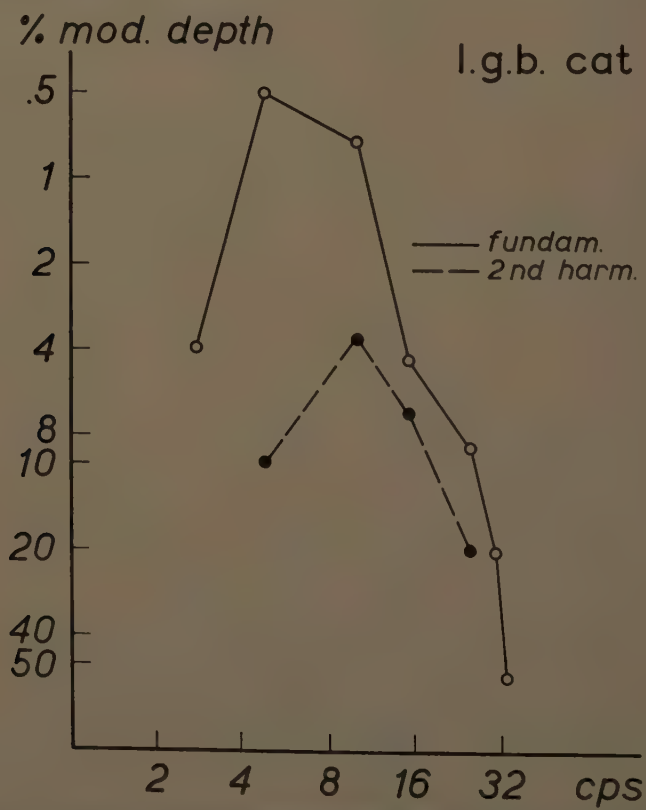

FIGURE 13. Bipolar recording in lateral geniculate body (l.g.b.) of curarized cat, using gross electrodes, modulated light at 15°, and 50,000 trolands. The curves represent the modulation needed for a constant response at the fundamental frequency of the modulation and for the second harmonic. Recorded with selective amplification (Q = 20). At low and high frequencies a steep attenuation is shown.

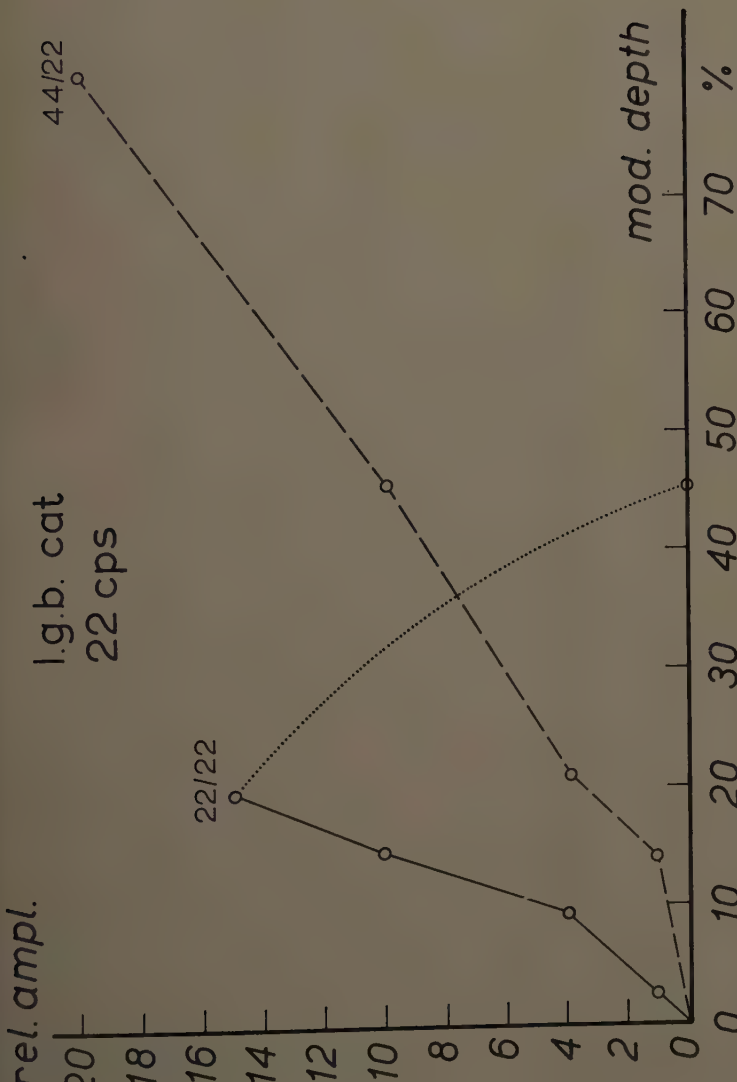

FIGURE 14. The technique used was that described for FIGURE 13. Here the amplitude of the fundamental and the double frequency is plotted as a function of the modulation. The response of the fundamental drops to zero at about 45 per cent of modulation. Up to 20 per cent the response is reasonably linear. There were no points measured between 20 and 45 per cent.

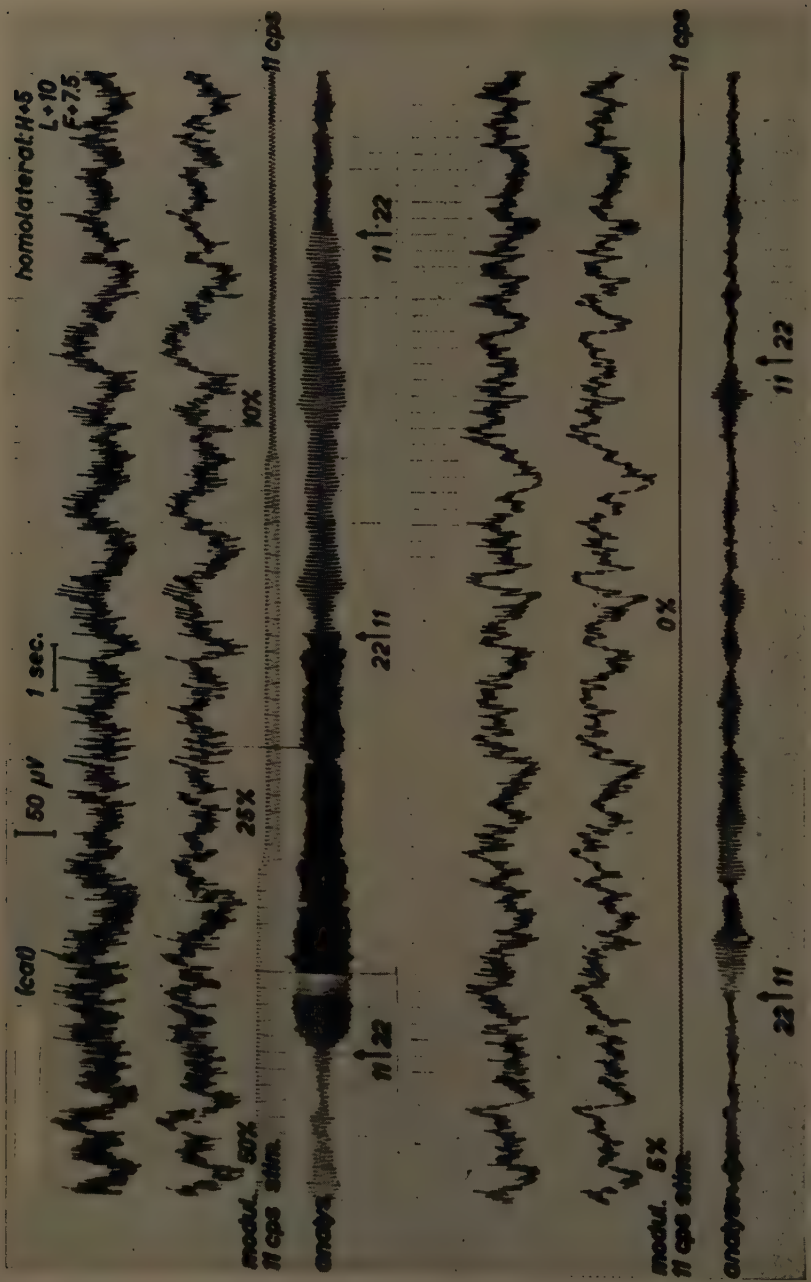

FIGURE 15. Recording of the electrical responses in the l.g.b. of the cat, using the same technique described for FIGURE 14: light diffuse on the eye ~200 lux. Curves 1 and 2 are direct identical recordings. Curve 3 shows the modulation signal. Curve 4 is the selectively-amplified response at the fundamental and double frequency. Decrease of the modulation depth gives a decrease of the second harmonic. The response at 10 per cent is much higher than that at 50 per cent for the fundamental frequency.

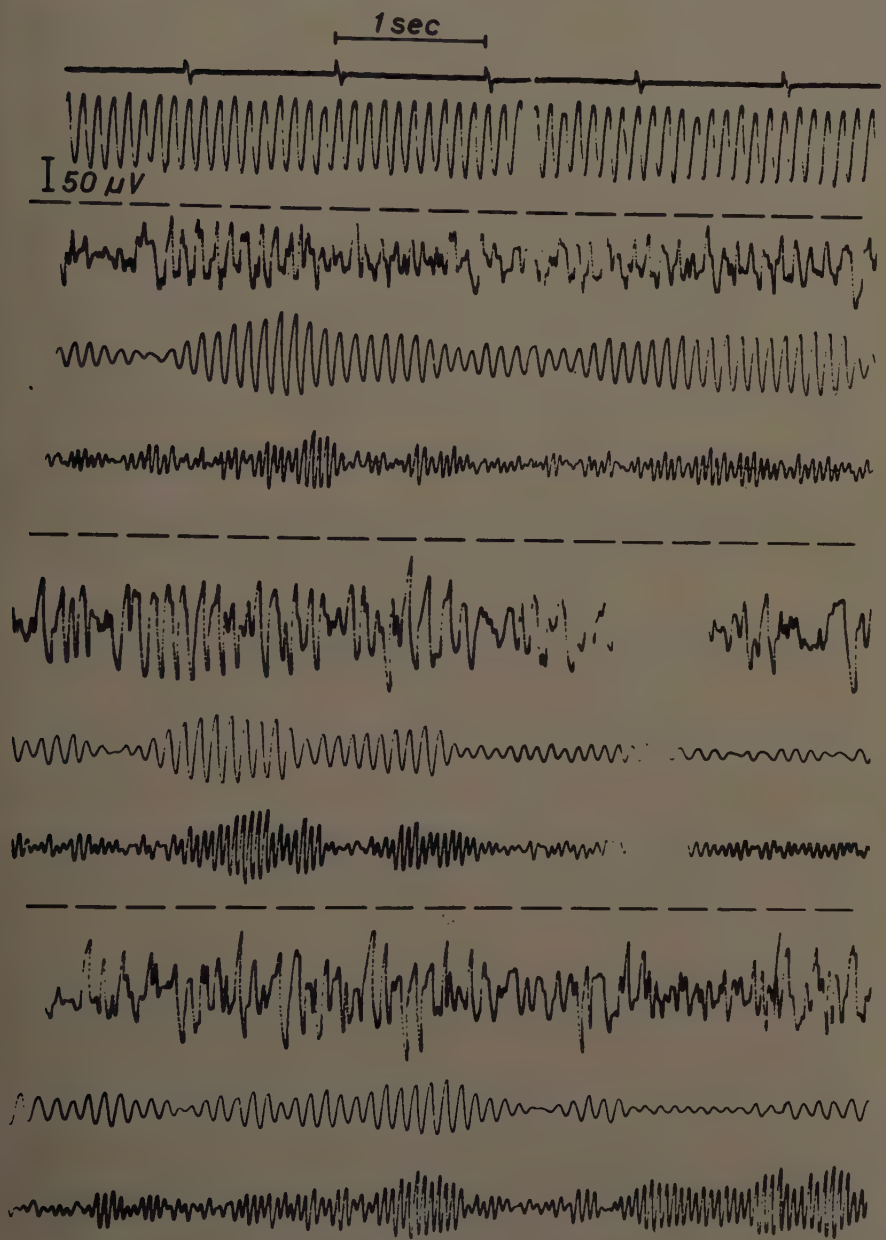

FIGURE 16. Simultaneous recordings of three leads, separated by about 10 mm., analyzed when stimulated with a frequency of 10 cps and a modulation depth of 50 per cent. The analysis has been made at 10 and 20 cps. Note the high fundamental response in the first channel, which occurs at a different time than the responses in the other channels. The second harmonic response is highest in the third channel, although fundamental responses are low. The second trace represents the modulation signal. Reproduced by permission from *Acta Physiologica Scandinavica* (Kamp *et al.*, 1960).

cortical leads. The electrodes were about 1 mm. long and 50 μ thick and separated by about 10 mm. It is seen that with 50 per cent modulation the responses of the second harmonic and those at the fundamental (10 cps) do not reach their maximum at the same time and at the same place. In FIGURE 17 an example is given of the response at 10 cps with a modulation depth of only 5 per cent. The CFF curves show a high attenuation below 8 cps and also above 25 cps. Objective curves of this patient and subjective of the author under comparable visual conditions are presented in the same FIGURE 2. Although the slopes will be a function of the noise spectrum, the steep fall at high and low frequencies cannot be expected to undergo large corrections for constant response curves. The responses were not very dependent on the intensities

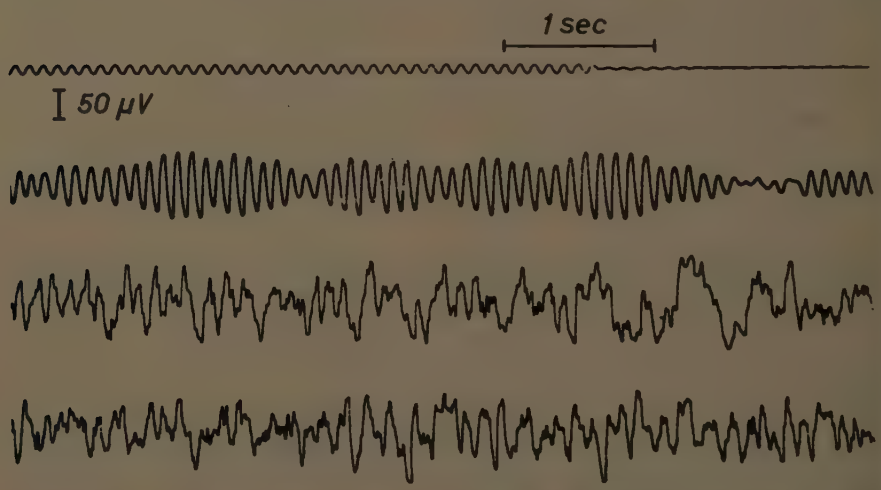

FIGURE 17. A clear response is shown in channel 1 with a modulation depth of only 5 per cent. The modulation happened not to be decreased instantly. The amplitude of the analyzed signal stays constant for a short time after the decrease in modulation depth. It is not improbable that this is still a real response. Reproduced by permission from *Acta Physiologica Scandinavica* (Kamp *et al.*, 1960).

used within a 25-fold range. Here also second harmonic responses could be found that were stronger than those expected according to a logarithmic law.

A number of experiments was also performed on two normal human subjects with integrating technique and scalp electrodes. FIGURE 18 shows a response at 44 cps, definitely below threshold, compared with the "noise" in the same channel. FIGURE 19 gives a response at 5 cps and 25 per cent modulation. The doubling of frequency shows that here, too, flicker processes exhibit a complicated structure, which has not been accounted for in our theoretical considerations.

Discussion of the Electrophysiological Material

If we accept the l.g.b. curves of the cat to have some importance also for man, we see that whereas in the ERG the high frequencies are not nearly as much attenuated as in perception, this is more the case in the l.g.b. and also

in responses from certain cortical regions in man. A high sensitivity in l.g.b. and cortex is found at about 10 cps. The responses are of a slow kind because they influence the selective amplifiers, which would not be the case if they were

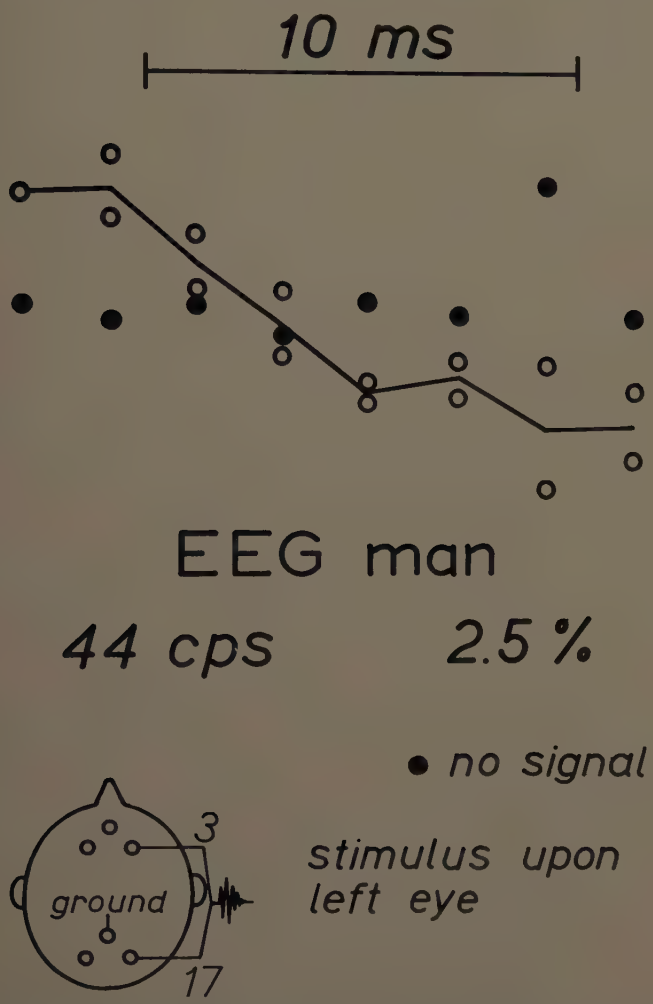

FIGURE 18. Scalp electrode recording in man with large visual field. Below CFF, 1000 integrations. Two runs are represented. There is a clear response. The circles show the signal given when light was interrupted. Channel 7 of the integrator was apparently maladjusted.

composed of action potentials. They may represent reasonably well the sinusoidal input signal as does the ERG. From the latter it is clear that the phase shifts correspond with certain features of the step response, which are not compatible with a minimum phase shift network. It might be useful to reconstruct incremental step responses from frequency response also in cortex and l.g.b.

Second harmonic responses found at comparatively low modulation percentages (but until now not below subjective threshold and tending to decrease with the modulation depth) show that in cat and man also essential nonlinear functioning occurs. The picture of flicker fusion is complicated by the fact that the amplitude characteristics at different points within the same anatomical region may differ substantially, especially below 5 cps, at which frequencies

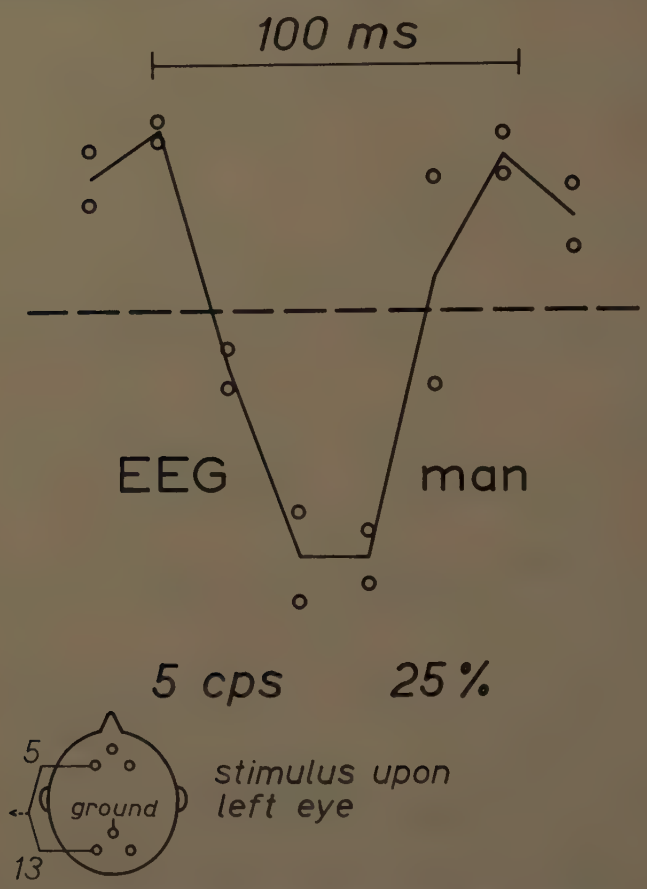

FIGURE 19. The recording conditions are similar to those used in FIGURE 18, but there is flickering subjectively. At 5 cps there is doubling of the presented frequency.

the human eye is still very receptive to flicker. In a number of cases, only very low responses have been found in these frequency regions; with integration, however, some responses at 3 cps were found with scalp electrodes.

We have presented these incomplete results to show the possibilities and difficulties of the method. A summary may be presented:

Sinusoidal modulation of light is represented in different parts of the visual system. The transmission is more or less linear and is recorded in some cases even below threshold. The amplitude function of the ERG differs in many

aspects from the subjective one. Some l.g.b. and some intracortical curves have large attenuations at low frequencies. The amplitudes or "objective thresholds" are not much dependent on the intensity, which could be explained by logarithmic functioning. Nonlinearities although present are not prominent in the ERG, but in the l.g.b. and cortex they are certainly stronger than can be explained by a logarithmic function. The measured representations are not the action potentials themselves but apparently a not-too-distorted image of their frequency.

The complicated character of low-frequency responses leaves the question open as to whether flicker is represented in more than one way and also which responses reported in this work could be the adequate ones for flicker perception.

Final Comment

We are well aware that our treatment and part of our experiments are still rather crude; however, they offer a basis for discussion. Certainly important changes in our concepts will be needed when more material is available. We hope, however, to have shown that the method of linear approximation in electrophysiology may lead to a better understanding of flicker phenomena. The distinction of strong and weak signal experiments might have value also for other fields of physiology. Frequency modulation of repetitive stimuli might then take the place of the modulation of light in our experiments. It would seem that further analysis of electrophysiological responses in man would help to resolve the important problem of how quantum statistics influence our threshold decisions.

Acknowledgments

It is nearly impossible to mention everyone who has been of importance in preparing this manuscript. Although of course the responsibility remains with me, the whole work is a direct consequence of the "school" of biophysics started by the late H. L. De Vries in Holland. Discussions in the "sense physiology" group of the Stichting Biophysica, Utrecht, greatly influenced my work through the kind but critical analysis of problems that prevails there. I am especially grateful to M. A. Bouman, H. De Lange, and A. Vendrik. As is clear from the quoted literature, the collaboration with J. J. Denier van der Gon has been of the greatest value to me. J. Strackee also contributed much to the final result.

Most of the electrophysiological work was done with P. Visser. J. ten Cate gave us hospitality for our experiments. A. Rémond's great knowledge and technical skill were of great importance in the use of his integrator for parts of this work. C. W. Sem-Jacobsen and W. Storm van Leeuwen were especially helpful in the Oslo experiments. F. T. H. Veringa's critical discussions also are appreciated very much. As they have in the past, H. den Hartog and F. A. Muller devoted much time to my problems.

FIGURES 9, 10, 11, 12, 18, and 19 were recorded together with A. Rémond (Laboratoire EEG, Salpétrière, Paris, France) and Visser (Laboratory of Physiology, University of Amsterdam, the Netherlands). FIGURES 13 and 14 employ data derived by Van der Tweel *et al.*, 1958, and FIGURE 15 was recorded

with K. Mechelse and M. van Vliet (Neurology Clinics, University of Leiden, Leiden, the Netherlands) and Visser.

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Part IV. The Reticular Formation

ON CIRCUIT PATTERNS OF THE BRAIN STEM RETICULAR CORE

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Interaction of the biological organism with its environment depends on a steady flow of reasonably valid information about that environment, continuous comparative evaluations of that information in past and present reference frames, and the selection of suitable behavior modes. Reception, transmission, and meaningful registration of information are now known to constitute problems in theoretical and practical neurology undreamt of 20 years ago. The last two decades have shown conclusively that the presence of sophisticated receptors, adequate transmission systems, and suitable cortical arrays for reception and manipulation of the data are not sufficient.

Three concepts in particular have enriched our understanding of the way the brain works. The first of these was epitomized by Dusser de Barenne and McCulloch's formulation of continuous two-way circulation between thalamus and cortex as the basis for upper brain activity (1941). The second was the discovery of a nonspecific, extralemniscal system projecting widely upon the cortex in parallel with specific sensory paths suggested by Forbes and Morison (1939) and shown clearly in the experiments of Moruzzi and Magoun (1949) and others a decade later. The third was the interpretation of Claire and Bishop (1955) that dendrite shafts may function characteristically as decremental, nonpropagated conductors, thereby freeing them from the functional rigidity of all-or-none digital repeaters.

Although we still know virtually nothing of the biomechanics of human decision, we feel intuitively that, for the most part, decision is not likely to be the product of sudden flip-flop phenomena, as when relays open or close. Rather, it must be a progressively formulating "focussing down" from a larger series of possibilities to a smaller one, utilizing possibilities that are already to some extent individually weighted by the intrinsic structure of the situation and by the character structure and previous life experience of the individual involved. As the various possibilities are sifted, continuously fedback loops from both inside and outside of the organism may be visualized as adding emphasis and credibility to some and detracting from others. Such shaping interactions should occur at every synaptic station, so that the final output as it is achieved tends to resemble the stepwise emergence of a marble statue from the crude block rather than the sudden appearance of typed figures on an output tape.

If this analogy has validity, we must look for a system—or a system of systems—that is represented at every synaptic level along input and output channels and that, in turn, is continuously supplied with the widest possible spectrum of data from within and without the organism.

Accordingly, we concern ourselves with the nature and circuit arrangements of a system that appears to be arranged in parallel with the classical sensory channels, the brain stem reticular core. Much has been written about the connections and functions of this core (Henry Ford Hospital Symposium, 1958) and much remains to be learned. Time will allow us only to outline something of the internal organization of this system, and its possible role in the neural scheme.

The reticular core consists of a phylogenetically primitive (Kappers *et al.*, 1960) matrix of nerve cells, axons, and glia, extending from the lower end of the medulla oblongata to the anterior end of the brain stem, and then by means of as-yet-poorly-defined projections to paleo- and neocortex. Although this system does not show the highly organized architectonic stereotypes characteristic of cortical tissue, patterns of anatomical and functional organization are clearly definable (Scheibel *et al.*, 1955).

Input Arrangements

When a typical cross section of the lower brain stem is viewed, as revealed by the silver impregnation methods of Golgi (Ramon y Cajal, 1952), dendrite processes of reticular cells are seen to project in all directions, interdigitating massively, and even projecting, in some cases, into specific cranial nerve nuclei and fiber bundles. The extension of individual dendrites is appreciable, often approximating one half the cross-sectional area of the brain stem in mouse, rat, and kitten (Scheibel, 1951), and raises questions as to the ultimate significance of fine nuclear subdivisions such as suggested by Olszewski and Baxter (1954) and others.

When the well-impregnated sagittal section is studied, a geographical theme is immediately obvious. Dendrite shafts are seen to be arranged primarily perpendicular to the long axis of the brain stem, with a secondary and much less extensive system projecting rostrocaudally (Scheibel and Scheibel, 1958). This orientation mimics the two main directions in which axons course through the system. The rostrocaudal axis is followed by all efferent reticular axons that, as we shall see, run to higher and lower stations in the neuraxis, effecting continuous *en passage* contacts with reticular dendrites. In addition, most longitudinal fiber systems of the brain, both afferent and efferent, follow a course roughly parallel to this axis. The great majority of these systems also make contact with reticular cells, either by collaterals or through terminals that turn at approximately right angles to the parent fibers, penetrate varying distances into the reticular core, and establish multiple synaptic contacts with reticular dendrites and cell bodies. Thus the two main orientations of reticular dendrites mirror the orientation of adjacent axonal components, and make for parallel axon-dendrite appositions, allowing for repetitive axodendritic synapses.

If the patterns of ascending and descending fibers projecting as collaterals or terminals upon the reticular core are studied, extensive degrees of overlap are seen. Similarly, if one studies the details of the synaptic scale upon the individual cell (Lorente de Nó, 1938), the degree of convergence of heterogenous afferents upon the single unit is impressive (Scheibel *et al.*, 1955). Histological and microelectrophysiological analyses show that presynaptic

termini from as many as one-half dozen or more separate fiber systems may contribute to the terminal array on any one cell. Similarly, the array on each cell tends to differ somewhat from the next. Thus each cell becomes an integrating subcenter with its own unique properties, and the whole reticular core becomes a mosaic of units whose individual outputs can be arranged along a spectrum of possibilities.

Thus far we have discussed only the direct transsynaptic communication of afferent impulses upon the reticular neuron. However, in the immediate neighborhood of each reticular soma are hundreds of dendrite shafts of near and distant cells. Processes of the former may be quite large, of the order of 2 to 5 μ, as they skirt the reticular element in question. Processes from more distant units may be finer than 0.5 μ in size. In addition to these dendrite processes passing near the soma, there may be tens of thousands of axons in the vicinity, some myelinated, some unmyelinated, but all presumably carrying impulses at one time or another. In the case of the axons, most are probably node-bearing, and it is now quite generally accepted that in such fibers, transmission is saltatory (Tasaki, 1953). Our individual reticular unit is thus surrounded by a dense matrix of electrically active tissue elements; some are the site of decrementing fields collapsing along their surfaces and others support propagated disturbances leaping along their surfaces several hundred μ at a time from node to node, with ion flow lines presumably extending considerably beyond the axon surface. Neurophysiological opinion remains divided on the possibility for cross talk among such units in the nonpathological state. However, there is sufficient evidence in the literature to indicate that, at least under some conditions, nonsynaptic (ephaptic) interaction can be demonstrated (Arvanitaki and Fessard, 1936; Arvanitaki, 1942; and others). Mountcastle (1957) and Mountcastle *et al.* (1957) have estimated the diameter of fields surrounding active cortical cells as of the order of 100 μ or more, and assume that "... it seems possible that an active cortical neuron will influence its neighbors by ephaptic interaction."

In so far as we know, similar measurements have not been made for reticular neurons. However, in a previous study (Scheibel and Scheibel, 1958), calculations were made in an attempt to determine to what extent the activity of the individual reticular neuron might be modulated by adjacent elements. Each soma-dendrite sphere, because of the dendrite orientation previously mentioned, was assumed to be flattened on its rostrocaudal faces until it achieved the dimensions of a cylinder of 100 μ in height (the rostrocaudal dimension) and 200 μ in diameter (a conservative estimate of the average length of a single reticular dendrite in the brain of a 10-day-old kitten). Average cell density in the lower two thirds of the reticular tegmentum of such an animal was calculated as 33 cells per 10^6 cubic microns. In the soma-dendrite envelope thus described, and called, for brevity, $a.\ p.\ i._{(sd)}$ (area of potential interaction of the soma-dendrite complex), we calculated that 4125 cells would be present. It is therefore theoretically possible that a cell aggregate of this number might show modifications in their own activity patterns or firing thresholds following discrete activity in one cell.

In cortical tissue, it seems unlikely that a single cell would be activated by an afferent barrage due to the extensive arborization of most cortical presyn-

aptic afferents. In the reticular formation where presynaptic termini are simpler, when the extracellular microelectrode was sampled from two adjacent reticular elements simultaneously, we have often seen one unit driven by an input that has no effect, or even an opposite effect, on the other. Direct transsynaptic effects probably supercede ephaptic ones, but in those cases where one of the pair alone receives contributions from the active fiber, field potentials in the surround might become meaningful and the number of elements thereby affected might become significant.

Output Arrangements

Terminal degeneration studies (Nauta and Kuypers, 1958; Brodal, 1957) can only hint at the extensive elaborations of the individual reticular axon emitted by a single cell. Golgi silver impregnations in suitable thick sections indicate that most reticular cells emit axons that run for long distances rostrad or caudad, emitting frequent collaterals as they go. The larger cells, such as those of the magnocellular group in the bulb, and many of the more mesially placed elements in the pontile and mesencephalic tegmenta are characterized by axons that bifurcate into rostral and caudal coursing segments. The total lengths achieved by such axons may be quite remarkable. Thus a cell of the pontile tegmentum may send its rostral segment through the mesencephalon, the entire length of the hypothalamus and into the basal olfactory region and septum. The caudal segment may enter the spinal cord in the ventral or lateral funiculus, terminating on or about internuncial or dorsal horn cells at a number of levels. Along the entire course of the axon, collaterals are given off to adjacent portions of the reticular formation, to cranial nerve nuclei, and to other sites (Scheibel, 1955; Scheibel and Scheibel, 1958). Because of the complexities of this system, it seems preferable to handle each system of projections in turn.

The collateral systems. The number and variety of side branches that the reticular axon may give off is surprising. Some are little more than a bouton on a stalk of a few μ in length, effecting contact with cells (neurons or oligoglia) virtually in the path of the main conductor. Others are longer, leaving the parent fiber at right angles to wander some distance into the reticular matrix, making synaptic contact with more remotely positioned cells. Characteristic also of such fibers are the side branches that enter nonreticular nuclei along the way, that is, sensory and motor cranial nerve nuclei and extrapyramidal centers.

In an effort to make some estimate of the degree of interaction of these fibers among the surrounding reticular population, a number of measurements were made of collateral length, direction, and frequency of branching from the main radical in kitten material (Scheibel and Scheibel, 1958). Upon averaging the data, we were led to make some simplifying assumptions that enabled quasi-quantitative interpretations. We assumed that the typical reticular axon emits one collateral in some direction from the parent stalk every 100 μ. Each collateral is assumed to be 100 μ in length. We can then transcribe a circle of 100 μ radius whose volume we shall call the area of potential interaction of the axon ($a.\ p.\ i._{ax}$). Operating from the further assumption that every reticular cell within this volume may, although not necessarily,

be acted upon synaptically by the fiber in question or by its dependences, and still working with our previously-derived figure of 33 cells per 10^6 cubic μ for average cell density in the lower two thirds of the brain stem, we calculate that 27,500 cells may lie in the *a. p. i.*$_{ax}$ of a single fiber of a conservative 2-cm. length. On the basis of the average cross section of a 10-day-old kitten stem, it can be shown that such a cell population approximates 0.3 per cent of the total reticular cell population below the diencephalon. We infer that if 300 crucially placed reticular axons were simultaneously active, they would temporarily commandeer the activities of a reticular population of almost nine million cells.

The possibility that a fiber ensemble thus crucially arranged will be selected for activity at one time is improbable. Also there is reason to believe that when a spatiotemporal configuration traverses a system, reticular or otherwise, a number of parallel channels are in simultaneous use according to the nature of the configuration, its intensity, locality, and quality. The number of channels for simultaneous transmission of discrete bits of information are thus more widely restricted. In the section on input arrangements we noted how afferent systems overlap each other as they project upon the reticular core. From this point of view, too, competition for circuit space must be active. In addition to the restrictions already assumed to be present in this system, data from a recent intracellular recording program suggest still another (Scheibel, Uchiyama, Uchiyama, and Scheibel, work in preparation). In a series of experiments, we noticed that as the intracellular probe was advanced through reticular tissue, frequent shifts in resting potential were recorded. These were of an amplitude and direction that could be considered to mark the entrance, passage through, and exit from successive neural (and glial) components. Only one or two such penetrations per hundred resulted in the appearance of spontaneous or driven spike activity. Since ordinarily such a preponderance of nonneural elements (glia, vascular tissue, and others) would not be expected, our only alternative interpretation was that in a neuron pool such as the reticular formation, only a fraction of the available units are maintained at or near those levels of excitability compatible with transmission. The rest must exist as a reservoir of temporarily nontransmitting units available for use at other times. Operations of this sort would be in poor accord with usually accepted theories of "place" specificity in conduction systems and would suggest, instead, a far more plastic mode of transmission where originating locality of the bit was represented in other ways than by strict intrafascicular geography. Thus there is further gain in the over-all flexibility of the system and further loss in channel availability at any one time.

We are left to assume that ability to manipulate a number of inputs simultaneously is not a high-priority item for the reticular core. The stacking of heterogenous afferents upon the same cell body points the same way. It would seem, rather, that the system concerns itself with integrates of afferent and efferent data: a sample cross section of the specious present. The nature of the reticular output must then reflect the vectored nature of these inputs, both as to quantity and quality, and is continuously projected upon higher and lower stations in the neuraxis.

Before leaving the general area of collateral patterns, it seems indicated to

touch on the problem of "the Golgi type II component" of the reticular core. We have already called attention to the fact that in the reticular system there appear to be no true short-axoned cells. However, a significant proportion of the reticular components release one or more axon collaterals, close to the parent cell body, that break up in elaborate plexus arrangements similar to the termination patterns of short-axoned cells elsewhere in the nervous system (Scheibel and Scheibel, 1958). The path length of these collaterals is usually very short (50 to 200 μ) and the terminal elaborations are extensive. Potential theory would suggest these as ideal settings for the development of decrementing, nonpropagated fields. If so, then the very fact of activity in a reticular soma would immediately change field patterns in the immediate area of the soma, with consequent changes, in turn, on threshold values of this and adjacent cells. The significance of these changes is still unknown, but Fessard (1954) offers an excellent theoretical discussion of the properties of such circuits.

The descending systems. Axons from cells of bulbar, pontine, and mesencephalic levels can be seen in suitable preparations to stream caudally and enter the spinal cord. The organization of these systems was first studied by Papez (1926) and more recently by Neimer and Magoun (1948), among others. The methods of Golgi clearly show how intensively the collaterals and terminals of these fibers are synaptically involved at sensory relay stations such as the gracile and cuneate nuclei, and widely through the dorsal horn of the spinal cord. In these areas, the simple reticular presynaptic terminals appear to course obliquely through the sensory receptive fields made up of the characteristic arborizing and overlapping sensory bushes and postsynaptic cell ensembles, effecting contacts here and there, and appearing to fractionate the field (Scheibel and Scheibel, 1960).

Reticular control can also be effected at even more peripheral stations, as suggested by the work of Granit (1955), in changing the firing rate of retinal cells through mesencephalic tegmental stimulation, and by Hernández-Peón (elsewhere in this monograph), who observed depression of auditory potentials in the cochlear nucleus following tegmental stimulation, although data from Dell's laboratory (as reported in these pages) casts some doubt upon the interpretation of these results. Muscle spindles are also under the control of this system, as the work of Eldred and Fujimori (1958) indicates.

The actual nature of the synapse in the internuncial and ventral horn areas is not yet satisfactorily worked out. However, there seems little reason to doubt that the pattern would follow that of reticular projections to other areas, that is, multiple axodendrite synaptic contacts terminating with or without bouton contributions to the synaptic scale of the soma itself. On the basis of our study of the reticular synapses in the cortex, we suspect that the important transactions occur at the axo-dendrite level.

The ascending systems. In many respects, the ascending components of the system are the most crucial and the most difficult to understand. As pointed out by Nauta and Kuypers (1958) the presence of ascending fiber systems in the reticular core was known at the turn of the century by Kohnstamm and his associates (1908). However, such data proved of little significance until the work of Forbes and Morison (1939) and of Moruzzi and

Magoun (1949) emphasized the importance of this projection in parallel with the classical sensory systems. Recent investigations clearly indicate that an ascending axonal stream from reticular cells of bulb, pons, and mesencephalon can be traced at least as far rostrad as the anterior mesencephalic tegmentum. Here splitting occurs just behind the centromedian-parafascicular complex, one leaf projecting through the medial thalamus in the intralaminar zone while another descends ventrolaterally and continues rostrad in the zona incerta of the hypothalamus (M. Scheibel, 1955; Brodal, 1957; Nauta and Kuypers, 1958; Scheibel and Scheibel, 1958). The ultimate fate of these fibers is still not clear, and the Golgi methods have thus far given us only partial answers. The ventral component collateralizes widely in the incertal region and many fibers pass as far rostrad as the septum, basal olfactory area, and striatum. Because of the technical problems involved, we have not yet been able to establish whether any of these axons continue without synapse into the archicortex and neocortex. A number of them seem almost certain to collateralize or terminate in the ventral portion of the nucleus reticularis thalami and possibly the ventral anterior nucleus of the thalamus, while others are lost among the masses of preoptic and supraoptic hypothalamic cells.

Fibers of the dorsal component run anteriorly through the intralaminar system of nuclei, collateralizing widely into the adjacent sensory relay and associative nuclei of both sides. Many of them terminate in the anterior thalamic nuclei, or nucleus reticularis thalami, but some seem to perforate these and project at least into the striatum. Once again, the total extent of these projections is not known.

A second nonspecific system is clearly shown by fortunate Golgi sections to originate in the centromedian-parafascicular complex, run forward in parallel with the former system on its medial side, and collateralize, even more abundantly, to almost every thalamic site. This upper reticular component appears to be the one found by Sharpless and Jasper (1957) to mediate phasic components of consciousness in contradistinction to the tonic control exercized by the lower component. Once again, the rostral limits of this upper system remain to be worked out, but we are certain that many of these fibers collateralize and/or terminate in the ventral-anterior nucleus and nucleus reticularis thalami. It does not seem unlikely that axons of this system may continue rostrally as reticulocortical components.

Due to the contributions of both of these systems, and to the rich collateral addition from descending corticoreticular fibers, a heavy neuropil plate is generated in the nucleus reticularis thalami surrounding dorsal, ventral, and rostral aspects of the thalamus and in the adjacent anterior thalamic nuclear complex. Through this plate and the external medullary lamina applied to the inner surface of the internal capsule pass virtually all the thalamocortical fibers and most of the corticothalamic reflux. We have already suggested the analogy between this and the screen grid in a vacuum tube (Scheibel and Scheibel, 1960), thus exerting possible modulatory effects on all commerce between diencephalon and cortex.

We outlined the probable nature of the reticulocortical relay in previous publications (Scheibel and Scheibel, 1958, 1960). It seems likely that the nonspecific fibers terminate along the apical dendrites of cortical pyramids in

long-climbing axodendrite appositions. In view of the work of Claire and Bishop (1955), supporting a nondigital nature for cortical dendrite electrical phenomena, they may serve as bias adjustors, continuously resetting the excitability thresholds of cortical somata and initial segments. Since the nonspecific relay serves as a principal contributor of presynaptic endings to these shafts, it would seem that the reticular core exerts an essentially analogical control over the digital (all-or-none) output sequences of the cortical cells.

In addition to controlling cell excitability levels, it was suggested also that these fibers might be involved in organizing somatotopic representation at the cortical surface. It was found that repeated subgriseal bifurcations of the nonspecific fibers caused each to send a number of fibers toward the surface, so arranged in relation to each other as to form a cylinder whose height was the thickness of cortex and whose diameter was of the order of 1 mm. (Scheibel and Scheibel, 1958). This was also the figure given by Mountcastle (1957), who found that individual cortical neurons within such a radius usually responded to driving from only one point in the periphery. We suggested that this type of topical specificity might be due to reticular modulation of the cortical cell threshold to one specific area rather than another. Certainly the overlapping of sensory afferents was sufficient to cause wider driving effects than this.

Our knowledge of the structural organization of the reticular core, incomplete as it is, leaves us with an image with which it is possible at least to work. We conceive of a system whose input-output arrangements allow it to lie athwart all incoming and outgoing systems of the brain. Through every moment in time it receives continuous samples of information being relayed in these systems, and this information is fed heterogenously into the cell-fiber matrix of the core. While qualitative specificity must be lost in the integrative processes that occur, it is suggested that quantity and change of influx are important parameters in determining the output of the system. While the system seems arranged to vector its inputs so that its output is always a function of them severally, it is also known to be extremely fickle, adapting to a stimulus as it becomes more familiar. Thus, competition for input supremacy in the core must rest on more than the vagaries of local geography and the laws of statistics. The fresh and the unfamiliar are more than likely to usurp reticular activity from the stale and predictable. The less familiar (hence less predictable) the sensory or motor sequence, the more its outcome may matter to the organism. In fact, the reticular core seems primarily concerned with the problem of mattering, or the investment of a neural event with significance, without which, no matter how available the circuitry, the event is apparently neither experienced nor remembered.

The unique properties of this system, some already demonstrated, others inferred, appear to deserve special emphasis in a monograph on human decisions. Although the very decision to include this system may in itself finally appear to have been in error, it would serve to underline a basic dynamic of human decision making: continuous formulation and shaping as the problem is engaged and progressive redefinition and focussing down as further information modulates that already being processed.

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THE EFFERENT CONTROL OF AFFERENT SIGNALS ENTERING THE CENTRAL NERVOUS SYSTEM

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The centrifugal control of afferent transmission in central sensory stations, long suspected by the perceptive neuroanatomists Cajal,[5] Brouwer,[3] and Wallenberg,[42] is now firmly established as the result of numerous experiments carried out during the last six years.

Early as well as recent anatomical studies,[2,3,5,6,33,34,40,41,42] have shown in the central nervous system the presence of efferent or centrifugal fibers projecting down to sensory synapses as low as those of the spinal cord, of the spinal fifth sensory nucleus, of the dorsal column nuclei, of the cochlear nucleus, of the retina, and of the olfactory bulb. It has also been demonstrated in acute experiments that electrical stimulation of these fibers elicits inhibitory[11,32,10] or facilitatory[11,26,31,10] interactions with the ascending afferent impulses in all the above mentioned sensory centers (see the reviews on this subject by Hagbarth[13] and by Hernández-Peón[16]).

Furthermore, some descending influences that control synaptic afferent transmission act tonically during wakefulness. It has been shown that synchronized afferent volleys recorded from the lowest levels of the somatic, auditory, and olfactory central pathways are enhanced by anesthetics and by brain stem lesions, which impair consciousness.[14,29,19,27] These experimental observations are consistent with older observations in which larger cortical sensory-evoked potentials were recorded during deep rather than during light anesthesia.[8] Therefore the conclusion is warranted that the reticular activity related to wakefulness normally regulates the passage of a great number of sensory impulses to the central nervous system. Undoubtedly this new neurophysiological concept can account, at least in part, for the well-known psychological phenomenon concerning the constantly limited span of perception.

Since perception is invariably associated with the process of attention, it occurred to one of us (R.H.-P.) that sensory impulses generated by stimuli outside of the span of attention would be prevented from entering the brain by descending inhibitory influences such as those mentioned above. Correlated behavioral and electrophysiological studies carried out in unanesthetized cats with electrodes permanently implanted in the lowest subcortical sensory relays have substantiated this hypothesis. By recording synchronized afferent volleys evoked by brief sensory stimuli, Hernández-Peón and his co-workers[17] have demonstrated that the corresponding evoked potentials are reduced or abolished when the animal attends to stimuli more significant than the test stimulus. This phenomenon has been observed in the dorsal cochlear nucleus,[28] in the axons of the retinal ganglion cells along the optic tract,[22,23,37] in the spinal fifth sensory nucleus,[7] in the lateral columns of the spinal cord,[4] and in the olfactory bulb.[19] A similar reduction of sensory-evoked potentials at those lower stations of the mentioned sensory pathways has also been observed by Hernández-Peón and his co-workers[15] consecutive to monotonous repetition

of the same sensory stimulus. This phenomenon represents an electrical sign of the learning process, termed by Dodge "habituation," and we shall refer to it as afferent neuronal habituation. In all the former experiments the recordings were made exclusively from one sensory pathway at a time; therefore only the behavioral response of the animal to other stimuli was observed. In order to obtain a more complete view of how the waking brain controls the entrance of the various signals coming from sensory receptors, we decided to record simultaneously the spontaneous and the evoked electrical activity from the dorsal cochlear nucleus, from the optic tract, from the olfactory bulb, and from the spinal fifth sensory nucleus, as illustrated in FIGURE 1.

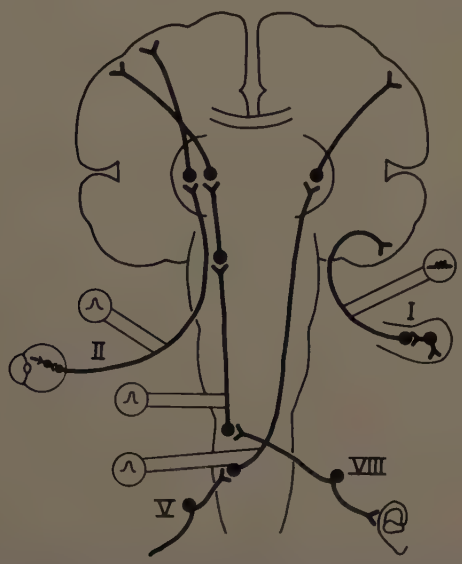

FIGURE 1. Diagram showing the recording sites from lower-order sensory neurons in the olfactory (I), visual (II), trigeminal (V), and auditory (VIII) pathways.

Material and Methods

The electrodes were stereotaxically implanted, as described elsewhere,[26,27] and the recordings were made with an ink-writing electroencephalograph. The test stimuli consisted of clicks, flashes of light, single electrical shocks delivered to the skin of the face, and puffs of air conveying fish odor. The different stimuli were delivered simultaneously or alternatively in patterns to be described later. Unless otherwise stated, the intensity of the test stimuli was kept constant throughout the entire experiment.

Results

Effects of alerting stimuli upon the electrical activity of lower order sensory neurons. The outstanding feature of the changes observed in the spontaneous and the evoked electrical activity in all lower sensory stations during alertness induced by different kinds of sensory stimulation was the great variability of

that activity as compared to the very stable evoked responses recorded from the relaxed animal. FIGURE 2 illustrates this effect during simultaneous records taken from the olfactory bulb, from the trigeminal nucleus, and from the cochlear nucleus. When the cat was awake but relaxed, the electrical activity of the olfactory bulb was minimal and uniform, and the tactile and auditory evoked potentials were approximately constant in size. When a mouse was

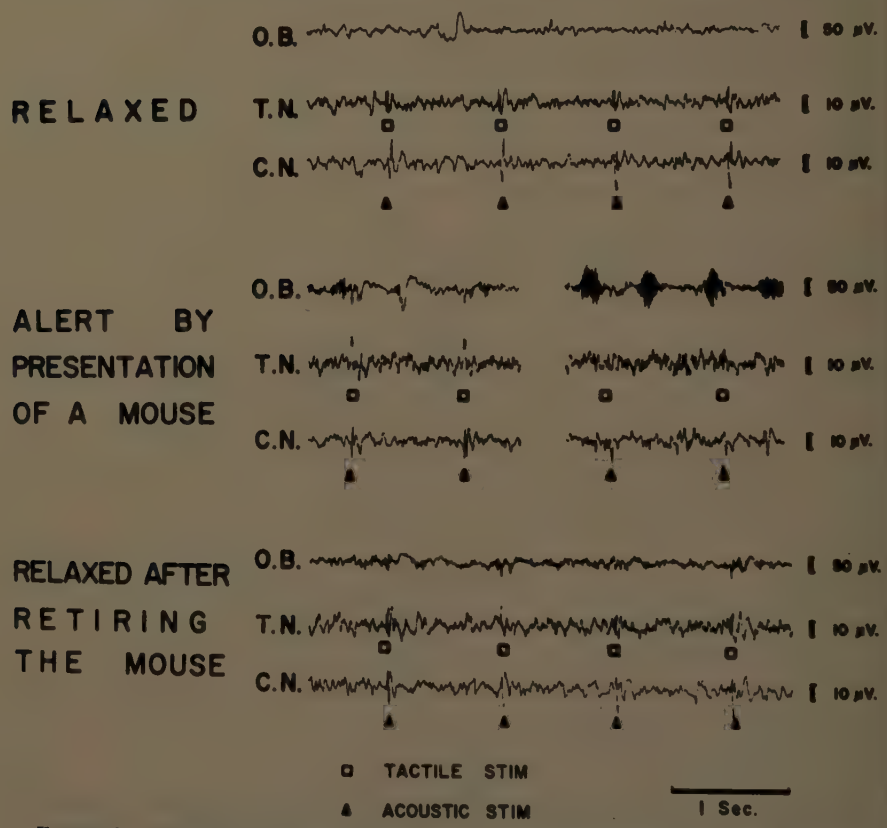

FIGURE 2. Changes in the electrical activity of the olfactory bulb (O.B.), the trigeminal nucleus (T.N.), and the cochlear nucleus (C.N.) in a cat during alertness induced by presentation of a mouse. Tactile potentials were evoked by mild electrical shocks applied to the face, and auditory potentials were evoked by clicks.

presented to the cat, significant changes of the electrical activity in those sensory structures occurred during the behavioral alertness of the animal. At first the auditory potentials diminished in amplitude, the tactile trigeminal potentials were facilitated, and bursts of rhythmic activity as described by Lavín *et al.*[35] and by Hernández-Peón *et al.*[26] began to appear in the olfactory bulb. A few seconds later, when the cat seemed to be attentively looking at the mouse, the tactile trigeminal potentials were practically abolished and the auditory potentials were further reduced, while the arousal discharges from the

olfactory bulb reached their maximum amplitude. Two minutes after removing the mouse from the cat, the electrical activity of the olfactory bulb, as well as the potentials recorded from the trigeminal nucleus and from the cochlear nucleus, regained their original amplitude. The reduction of the sensory-evoked potentials was maintained for a few seconds only as the cat appeared to be attentively looking at the mouse. As shown in FIGURE 3, when the cat was alert without being specifically attentive to a given object, the size of the evoked potentials varied intermittently from one moment to the next. These changes occurred simultaneously or alternatively in different sensory pathways.

Arousal reaction consecutive to cessation of repetitive stimulation and its manifestations in sensory pathways. An observation of everyday life is that arousal may be produced upon cessation of sleep-inducing monotonous stimulation. In order to study the electrophysiological correlates of poststimulatory arousal in sensory pathways, we decided to record the changes of the electrical activity in the various lower sensory synapses upon cessation of a series of monotonous clicks. The presentation of the first clicks induced the classical orienting response, and by repeating these clicks the animal became relaxed and drowsy. If at this moment the acoustic stimuli were suddenly stopped, the animal became aroused and high-voltage bursts of rhythmic activity appeared in the olfactory bulb. Poststimulatory arousal was rapidly habituated, as shown in FIGURE 4, just as the arousal reaction induced by presentation of a novel stimulus can be habituated.

The arousal response elicited by cessation of the monotonous series of clicks was no longer obtained when the procedure was repeated. It is evident, therefore, that habituation of poststimulatory arousal can be produced by one single cessation of intermittent stimuli.

Sometimes we also observed a transient reduction of the evoked potentials recorded from the trigeminal nucleus or from the optic tract during the first two or three seconds of poststimulatory arousal (FIGURE 5).

Afferent neuronal habituation in lower levels of the tactile, auditory, and visual pathways. The phenomena of habituation and dishabituation by diaphorostimulation already described in various sensory pathways[15] were readily confirmed in our experiments. In addition to recording the potentials elicited by the habituating stimulus, we recorded the dishabituating impulses producing disinhibition in another sensory pathway, that is, heterosensory hyperresponsiveness. FIGURE 6 shows that tactile habituation recorded from the spinal fifth sensory nucleus was easily disrupted by the sudden entrance of photic signals simultaneously recorded from the optic tract. Conversely, photic habituation recorded from the optic tract was released by sudden application of tactile stimuli (FIGURE 7).

Simultaneous recordings of evoked potentials from several afferent pathways showed different rates of habituation for the various sensory modalities. Tactile habituation always occurred faster than that elicited by acoustic or photic stimuli, as illustrated in FIGURE 8. After the establishment of photic habituation by monotonous repetition of flashes of light, suddenly applied tactile and acoustic stimuli resulted in the reappearance of the photically evoked poten-

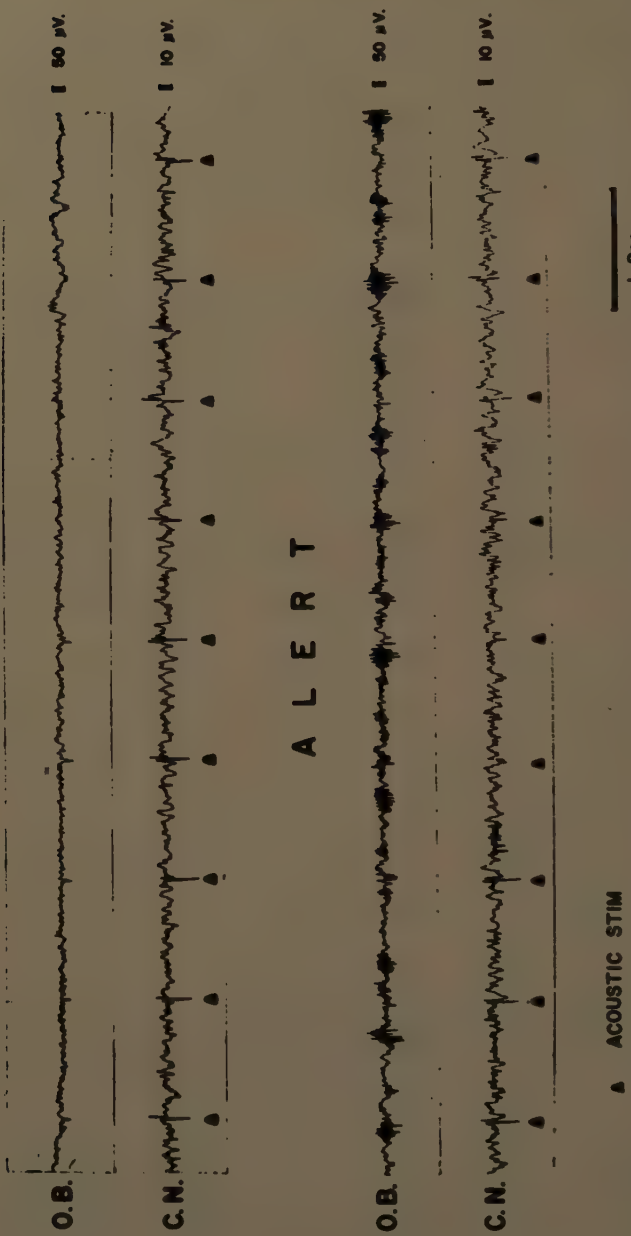

FIGURE 3. Electrical activity from the olfactory bulb (O.B.) and from the cochlear nucleus (C.N.), recorded from a cat during relaxed wakefulness and during alertness.

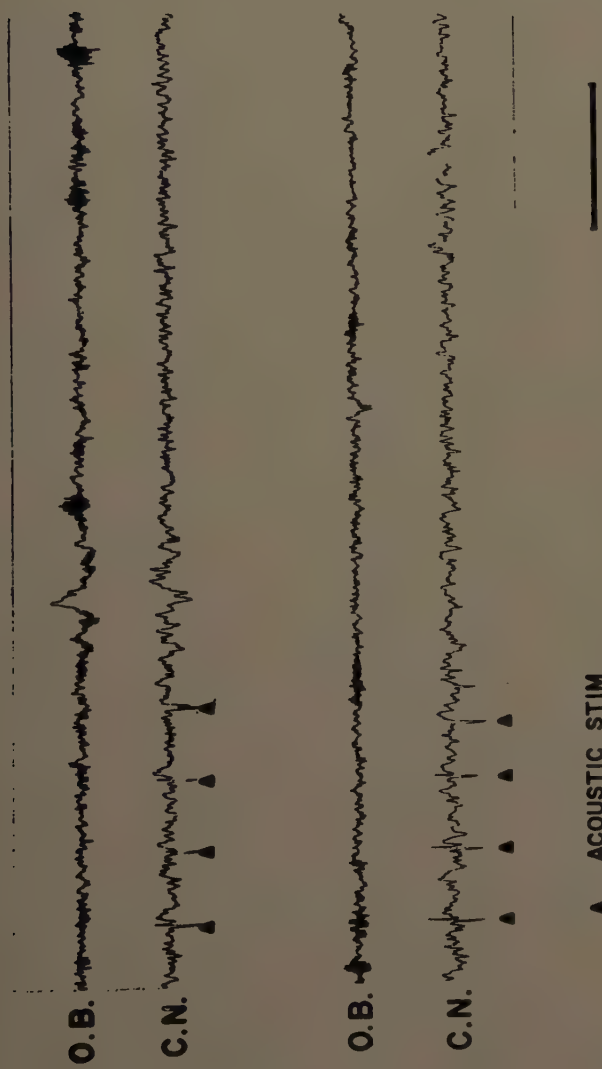

FIGURE 4. Changes in the electrical activity of the olfactory bulb (O.B.) and of the cochlear nucleus (C.N.), induced by cessation of a monotonous series of clicks, on two consecutive occasions.

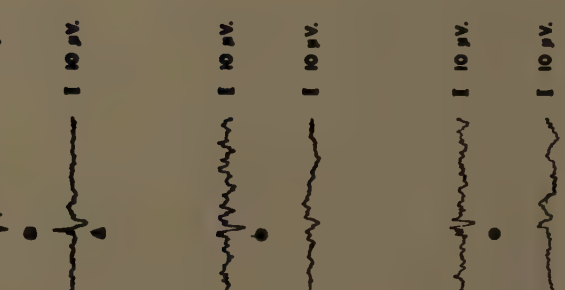

FIGURE 5. Changes in the photic-evoked potentials recorded from the optic tract (O.T.), following cessation of a monotonous series of clicks. Note a reduction of the photic potentials during six seconds of alertness after interrupting the acoustic stimuli.

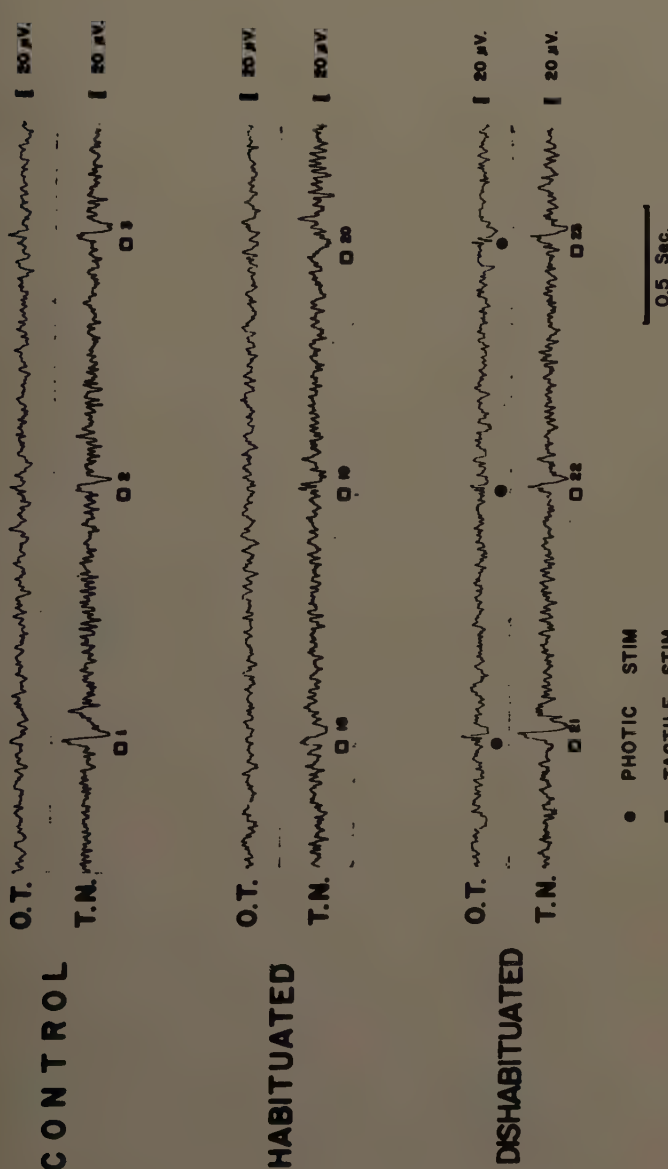

FIGURE 6. Habituation of tactile potentials recorded from the spinal fifth sensory nucleus (T.N.), and dishabituation induced by the sudden presentation of flashes of light that evoked potentials, recorded from the optic tract (O.T.).

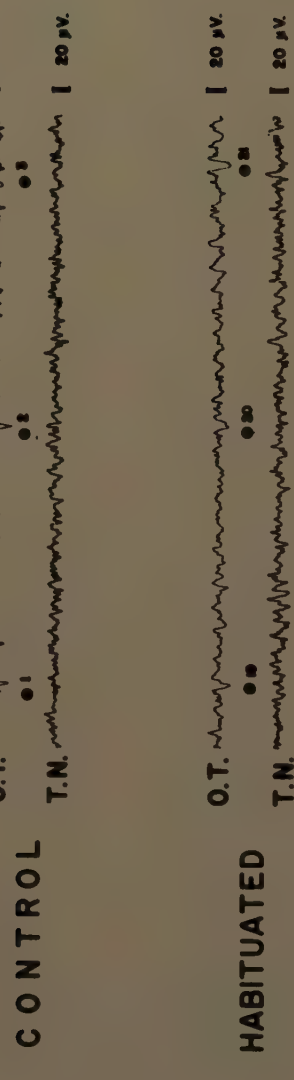

FIGURE 7. Habituation of photic potentials recorded from the optic tract (O.T.), and dishabituation by presentation of tactile stimuli, which evoked potentials recorded from the trigeminal nucleus (T.N.).

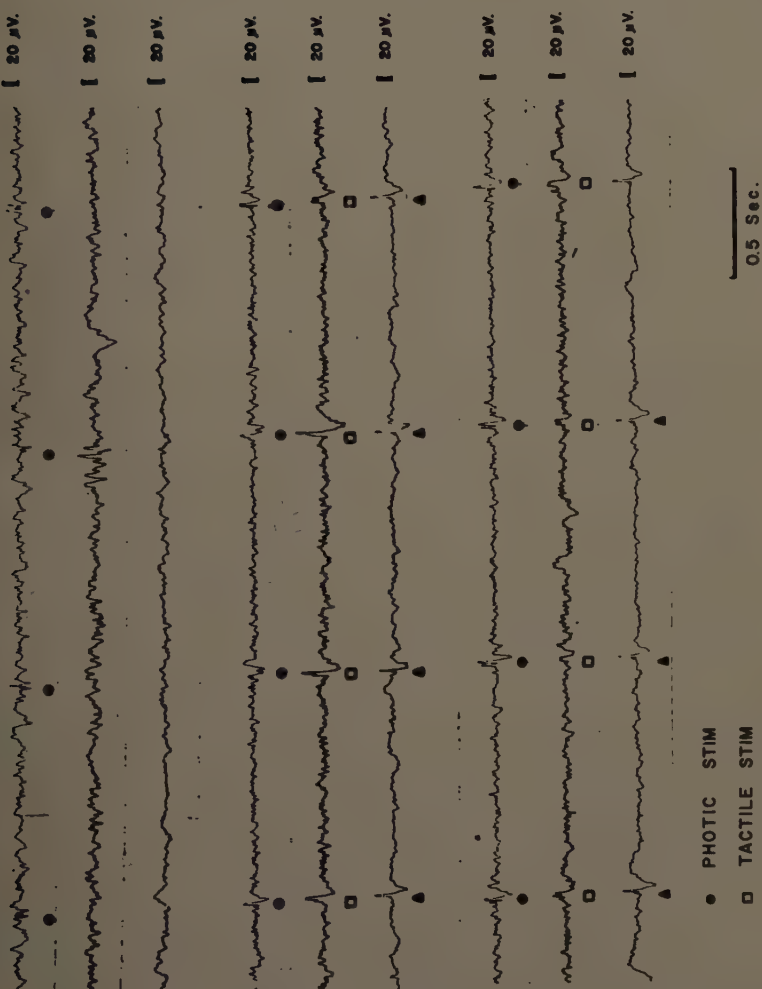

FIGURE 8. Simultaneous recordings from the optic tract (O.T.), from the spinal fifth sensory nucleus (T.N.), and from the cochlear nucleus (C.N.). Habituation of the photic potentials after 400 flashes of light is shown in *A*. *B* represents the restoration of the photic-evoked potentials elicited by giving tactile stimuli to the face of the animal, as well as clicks. *C* shows that after 20 tactile and auditory stimuli the trigeminal potentials were reduced, while the photic and auditory potentials remained unchanged.

tials in the optic tract. Shortly thereafter, during the simultaneous three-modality stimulation, the tactile potentials were significantly reduced in contrast with the photic and auditory potentials, which remained unchanged. Another typical effect is shown by the experiment illustrated in FIGURE 9, in which acoustic and tactile potentials were recorded simultaneously from the bulbar sensory relays. After a few dozen stimuli, the trigeminal potentials almost disappeared, whereas the auditory potentials remained unchanged. At this moment the application of a visual stimulus that alerted the cat brought about the reappearance of the trigeminal potentials (dishabituation), as well as a diminution of the auditory responses. In the experiment illustrated in FIGURE 2, the application of a given stimulus simultaneously elicited blocking of one sensory pathway and disinhibition of the other afferent system. The opposite effects are obviously related to the previous state of the sensory synapses involved.

Comments

Afferent transmission during alertness. The above mentioned experimental results provide direct evidence for the great versatility of the centrifugal systems that regulate sensory transmission during wakefulness. Facilitation of the electrical activity in one sensory pathway is always accompanied by blocking of sensory transmission in other afferent pathways, and this phenomenon occurs alternatively from one moment to the next in the various sensory systems. It seems that since the amount of information the waking brain can manage at any given moment is limited, a selection is accomplished by facilitating sensory transmission for the meaningful afferent signals and by simultaneous blocking of the remaining incoming signals to the central nervous system.

In discussing the neural mechanisms of alertness a distinction should be made between *unspecific alertness*, which often accompanies an inquisitive attitude on behalf of the animal, and *specific alertness*, characterized by a state of attention focused upon a given stimulus. In the first situation the descending or centrifugal influences that regulate sensory transmission act intermittently and in random fashion. As soon as attention is directed and focused upon something, the descending systems for sensory control maintain an organized activity.

In contrast with the great deal of variations observed during alertness in the electrical activity of sensory pathways, there is a great stability in the same sensory structures during relaxed wakefulness or sleep. These findings suggest that the centrifugal system involved in sensory regulation is related to the "arousal system" whose essential part is located in the mesodiencephalic region of the brain stem.

It may be argued that the apparently diffuse projections of the reticular system cannot account for the organized selection of sensory impulses during attention. However both anatomical and physiological studies have shown a complex organization in the brain stem reticular formation, which is not as diffusely arranged as originally thought. Sensory convergence in the different elements of that region is only partial[1,25] and the specific information received

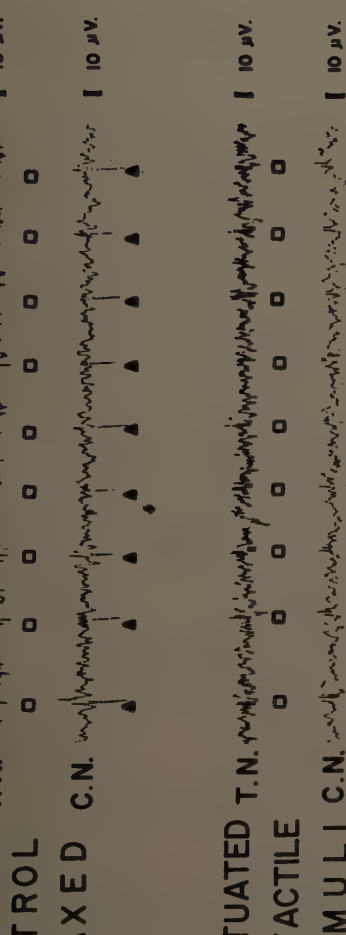

FIGURE 9. Tactile and auditory-evoked potentials recorded simultaneously from the spinal fifth sensory nucleus (T.N.) and from the cochlear nucleus (C.N.). The upper tracings show the evoked potentials at the beginning of the experiment. After 25 tactile and auditory stimuli (*middle tracings*), the tactile potentials were reduced while the auditory potentials remained unchanged. At this moment, a visual stimulus that alerted the cat (presence of the experimenter) brought about a reduction of the auditory potentials and an enhancement of the tactile potentials.

by the reticular neurons is preserved, as shown by their patterned activity evoked by the same sensory stimulus.[1]

As postulated in FIGURE 10, it seems likely that the feedback circuits linking cortical and reticular neurons to specific sensory elements are topographically organized so as to influence selectively the incoming sensory information. In addition to the corticoreticulo-sensory system, each specific afferent pathway is accompanied by a chain of neurons carrying impulses in the opposite direc-

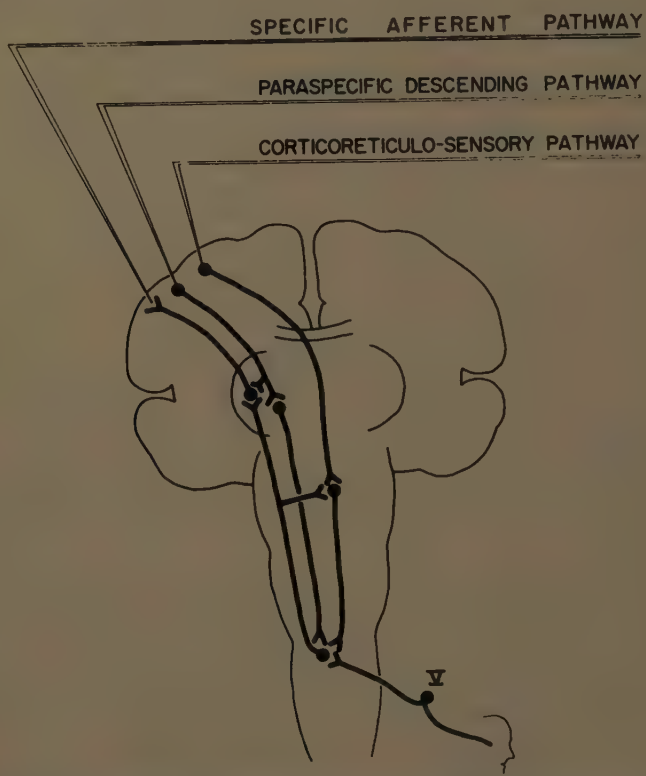

FIGURE 10. This diagram illustrates the descending pathways that can influence transmission at lower sensory synapses.

tion, which may be termed paraspecific descending pathways. The effects of electrical stimulation of the latter system are now being studied by C. Ajmone-Marsan (personal communication, 1960) and in several other laboratories.[32,36,38,9] Although there is yet no clue about the functional role of the extrareticular descending pathways, we venture to propose as a working hypothesis that the afferent control achieved by them might serve for fine sensory discrimination resulting from highly elaborated learning processes.

Poststimulatory arousal. Drowsiness or sleep induced by monotonous repetition of the same stimulus in a quiet environment is a common and intriguing observation whose mechanism remains obscure. Controversial interpretations

have been centered either on the reduction of stimuli, or on an active inhibitory process as the responsible factors for somnolence.

According to the first hypothesis translated into modern neurophysiological concepts, sleep would be the passive result of diminished activation of the brain stem arousal system, which is constantly impinged upon by a great variety of sensory impulses. The second hypothesis implies that rhythmic sensory stimulation progressively recruits inhibitory neurons acting upon the arousal system.

Cessation of monotonous stimulation provided an experimental procedure for testing the two above mentioned hypotheses. If sleep were passively determined, discontinuation of stimuli should enhance the depth of sleep. On the other hand, sudden interruption of inhibition usually elicits rebound excitatory effects that in this case would be manifested as arousal. Our experimental results are consistent with the latter hypothesis.

The recent discovery of an EEG synchronizing region caudally located in the brain stem also supports the view that sleep is the result of active inhibition. In experiments designed to test whether the bulbar synchronizing structures are responsible for the EEG synchronization induced by the sheer repetition of photic stimuli, Mancia *et al.*[39] have repeated this experiment in permanently awake cats with the brain stem transected at the midpontine level. These authors found that EEG synchronization can still be induced in the midpontine cats by applying a larger number of photic stimuli than those required in intact animals. Such experimental results indicate that in the intact brain the postulated bulbopontine inhibitory ascending reticular formation participates partly in the monotonously induced sleep. It seems likely that regular intermittent sensory stimulation also activates mesencephalic and supramesencephalic neurons, which are inhibitory for the mesodiencephalic elements of the arousal system.

There are suggestive data indicating that such sleep-inducing descending inhibitory influences might be mainly located within the limbic system. Both bulbopontine ascending and limbic descending inhibitory neurons, which would be the anatomical substratum of the "sleep system," were discussed with the problem of habituation of arousal by Hernández-Peón in the Moscow Colloquium.[15]

Plastic nature of habituation and dishabituation. Habituation and dishabituation occurring at the first sensory synapses have been discussed in detail from the neurophysiological standpoint in several papers.[15,24,27] The cumulative and long-lasting effects of repetitive sensory stimulation are manifestations of plastic inhibition, which must be distinguished from the transient sensory depression related to attention. Although dishabituation by diaphorostimulation can be interpreted as "inhibition of inhibition," it is also evident that the persistent effects of the dishabituating inhibition are not comparable to the transient depression produced by the same stimulus upon nonhabituated sensory-evoked potentials.

From our experiments it became clear that the same stimulus can elicit different effects in any given sensory pathway according to the previous experience of the animal. Therefore, if that experience is not taken into con-

sideration, dishabituation can easily be misinterpreted as simple sensory facilitation.

It can be concluded that the central nervous system is endowed with complex mechanisms that permit the selection of a limited amount of sensory signals long before they reach the higher integrating structures, and that this selection is made according to the significance of the stimuli and to the past experience of the organism.

Summary

In an attempt to study the simultaneous control of the various sensory impulses entering the waking brain, a series of experiments were performed on cats with bipolar electrodes permanently implanted in the olfactory bulb, in the optic tract, in the dorsal cochlear nucleus, and in the spinal fifth sensory nucleus. Simultaneous electrical recordings of the background and sensory-evoked activity from these structures provided the following results:

(1) In contrast with the rather stable magnitude of the various sensory-evoked potentials recorded during relaxed wakefulness, these potentials underwent a great deal of variation during alertness. When the cat became alert, the olfactory bulb activity and the trigeminal-evoked potentials increased, whereas the acoustic potentials diminished. Later, during focusing of attention, both the acoustic and the trigeminal potentials were reduced.

(2) Regular repetition of intermittent stimuli led to a progressive diminution of the evoked potentials in all sensory pathways (afferent neuronal habituation).

(3) The presentation of a stimulus that induced behavioral alertness brought about an enhancement of the already diminished evoked potentials (dishabituation) and a reduction of the larger potentials.

(4) Cessation of a monotonous series of clicks, which had induced drowsiness, brought about behavioral and electrical manifestations of arousal in the sensory pathways.

From the above-mentioned results it can be concluded that any given stimulus can elicit different changes in the lowest synapses of sensory pathways according to the significance of the stimulus and to the past experience of the animal.

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PATTERNS OF ACTIVITY OF SIMULTANEOUSLY RECORDED NEURONS IN MIDBRAIN RETICULAR FORMATION

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Introduction

The brain stem reticular formation is a particularly appropriate portion of the nervous system to discuss in this publication. Since the work of Magoun (1952), the reticular formation has come to be regarded as a major coordinating system between the afferent inflow and the efferent outflow. A general discussion of the neurophysiological literature related to the reticular formation is made unnecessary by the thorough reviews of Segundo (1956), Rossi and Zanchetti (1957), and O'Leary and Coben (1958). Our interest in the midbrain reticular formation stemmed from the observation that single reticular neurons often had wide peripheral receptive fields that extended over several limbs (Amassian and DeVito, 1954). However, altering the site of peripheral stimulation usually changed the pattern of evoked firing in a given reticular neuron. These experiments were done on cats under chloralose anesthesia. The analysis was subsequently extended by experiments on locally anesthetized cats (Amassian and Waller, 1958). It was proposed that different portions of the body were represented in the reticular formation by different temporal patterns of discharge of wide receptive field neurons. Attempts to separate the "position" code from the "intensity" code led to the conclusion that the distribution of activity within a group of reticular neurons following stimulation of one limb could not be matched by adjusting the intensity of a stimulus applied to another limb. Our attention was directed toward the possible importance of the intervals between the first discharges of different neurons as a mode of representation of the periphery. Further investigations required the simultaneous recording of the activities of several reticular neurons that were situated in a limited volume. A technique was recently developed for simultaneously recording from three or more cortical neurons by three independently manipulated electrodes (Amassian *et al.*, 1959). We inserted only two electrodes at a time into the midbrain reticular formation but aimed at recording two separable units with one electrode and at least one unit with the second electrode. Our analysis of the preliminary data was greatly facilitated by use of a simple, highly flexible computer whose over-all operation is described below.

Methods

Many of the experimental procedures were previously reported and require no further mention (Amassian and Waller, 1958). Briefly, cats were surgically prepared under ether anesthesia. Ether anesthesia was subsequently discontinued; the animal was paralyzed with Flaxedil and artificially ventilated with 100 per cent oxygen. The cut skin margins and deeper tissues were infiltrated with 1 per cent procaine hydrochloride. The micropipettes were filled with 3

M NaCl instead of 3 *M* KCl as in our previous experiments. One electrode was inserted in the vertical axis of the sterotaxic apparatus. The other electrode was slightly inclined so that the two electrode tips approached one another in the midbrain tegmentum. With practice, a unit recorded by one electrode can readily be held during a search for a unit with the other electrode. However, simultaneous manipulation of both electrodes is often required. At the termination of the experiment, a coarse micropipette was inserted into the lateral portion of the midbrain down to a convenient depth. The brain was subsequently fixed in 10 per cent Formalin in saline and frozen-sectioned at 40 μ. Sections were stained with cresyl violet. The bottom of the track produced by the crude "marker" pipette was used as a reference point in localizing the position of the tip of the microelectrode (cf. Amassian and DeVito, 1954). The simultaneously recorded units discussed below were recorded in the midbrain reticular formation and were situated about 2 to 4 mm. from the midline. The units were dispersed vertically between the level of the aqueduct and the upper border of the red nucleus.

Analysis

Neuronal activity was recorded on multichannel magnetic tape, using the microelectrode techniques described above and conventional amplifying equipment. On playback of this tape, the unit discharges were amplified, filtered, and center-clipped to remove base-line noise and fluctuations. The spikes were then employed to trigger standard pulses of 1-msec. duration and constant amplitude. Selection and suppression or canceling devices made it possible to produce a separate train of standard pulses for each unit recorded, even when two units were picked up by the same microelectrode.

These standard pulses were then passed to a logic unit, and the pulses that the logic unit allowed to pass were counted by one of a battery of electronic counters. The logic unit generated various functions related to the spikes or to the stimuli and used these functions to select the pulses to be passed to the counter.

Interspike interval distributions were constructed by producing a gating pulse (the "extension gate") initiated by a spike. This positive pulse ended after a preset time unless another spike arrived before the expiration of the pulse. In this case, the timing was started again from this latter spike, thus extending the pulse. This positive pulse was used as a gate, allowing only those spikes to be counted whose preceding interspike interval exceeded the preset time. A series of such runs with different gate settings produced the interspike interval distribution by subtraction.

A gate that allowed spikes to be counted only during a 10-msec. interval, occurring at an adjustable time after the stimulus ("traveling gate"), was used to produce the spike frequency distributions.

"Coincidences" were determined by a gate that allowed only those spikes on one channel to be counted that occurred within a fixed number of milliseconds after a spike on another channel. In this paper a coincidence always has a number attached: coincidence within T milliseconds. FIGURES 4 and 5 show plots of coincidences observed as a function of the value of T.

The expected random coincidence value was calculated by dividing the total

time available for coincidence (t_c) during the run by the total duration of the run, and multiplying by the total number of discharges of the unit. The total time the gate was open (t_c) was directly determined by counting 1000/sec. clock pulses that were led through an identical gate. Superimposition of spikes from two units on the same recording channel was missed by the computer, but was detected on an oscilloscope, was insignificant as a source of error, and was therefore not included in FIGURE 4.

Spontaneous Activity

Many midbrain reticular neurons are tonically active, but the intervals between discharges are irregular (Amassian and DeVito, 1954; Amassian and Waller, 1958). Using the extension gate, the distribution of intervals was determined during spontaneous activity. One kind of distribution is shown in FIGURE 1*b*. Note that most intervals are brief; 94 per cent are less than 200 msec. Although intervals longer than 60 msec. are progressively less common, their incidence does not show an exponential decline, and the incidence of intervals briefer than 40 msec. shows a sharp decline. If events are assumed to be randomly and independently distributed in time, so that the probability of one event occurring in an interval of time dt approaches $\lambda\, dt$ for some constant λ, as dt approaches 0, then the distribution of intervals between events can be shown to be an exponential function, and the events themselves will have a Poisson distribution (Feller, 1950). This assumption of independence of course makes no allowance for refractory periods; in fact the exponential distribution of interspike intervals shows that the extremely brief intervals are most probable. The assumptions given above can be modified to include an exponential recovery of responsiveness for the recorded unit, after firing, or for the excitation pathway (cf. Martin and Branch, 1958). The results of such a calculation are given in FIGURE 2*a*; this shows that even the assumption of a rather prolonged exponential recovery does not yield an interspike interval distribution that matches the observed distribution. However, if a sigmoid recovery curve is assumed, as in FIGURE 2*b* the resulting interval distribution is much closer to the observed. The majority of midbrain reticular units are not monosynaptically related to the afferent inflow. A sequence of relays, each assumed to have a short absolutely unresponsive period followed by an exponential recovery, can produce an over-all recovery curve for the sequence that resembles the sigmoid curve used in FIGURE 2*b*. Attempts to fit the observed distributions accurately are of limited value at present, since so little is known of the recovery of excitability of reticular synapses. The data presented here, however, show that even the irregular spontaneous discharges cannot be assumed to be independent random events.

A unit recorded simultaneously with the unit of FIGURE 1 yielded a quite different distribution of interspike intervals (FIGURE 3*b*). The neuron discharged at a very low over-all frequency of 0.6/sec., more than 60 per cent of the intervals exceeded 200 msec., (cf. 6 per cent in FIGURE 1*b*) and 30 per cent of the intervals exceeded 1 sec. in duration. Nevertheless about 25 per cent of the intervals were less than 80 msec. The distribution of spikes is totally unlike a Poisson distribution, and the interspike interval distribution resembles neither an exponential curve nor the modified curves of FIGURE 2.

The distribution of intervals of both the units illustrated in FIGURES 1 and

3, was altered by stimulation of the ipsilateral forepaw. The shift toward a higher percentage of brief intervals is very obvious in FIGURE 3*a*. By contrast, in FIGURE 1 the percentage of intervals briefer than 40 msec. is markedly reduced by stimulation (these shifts presumably reflect facilitation and depres-

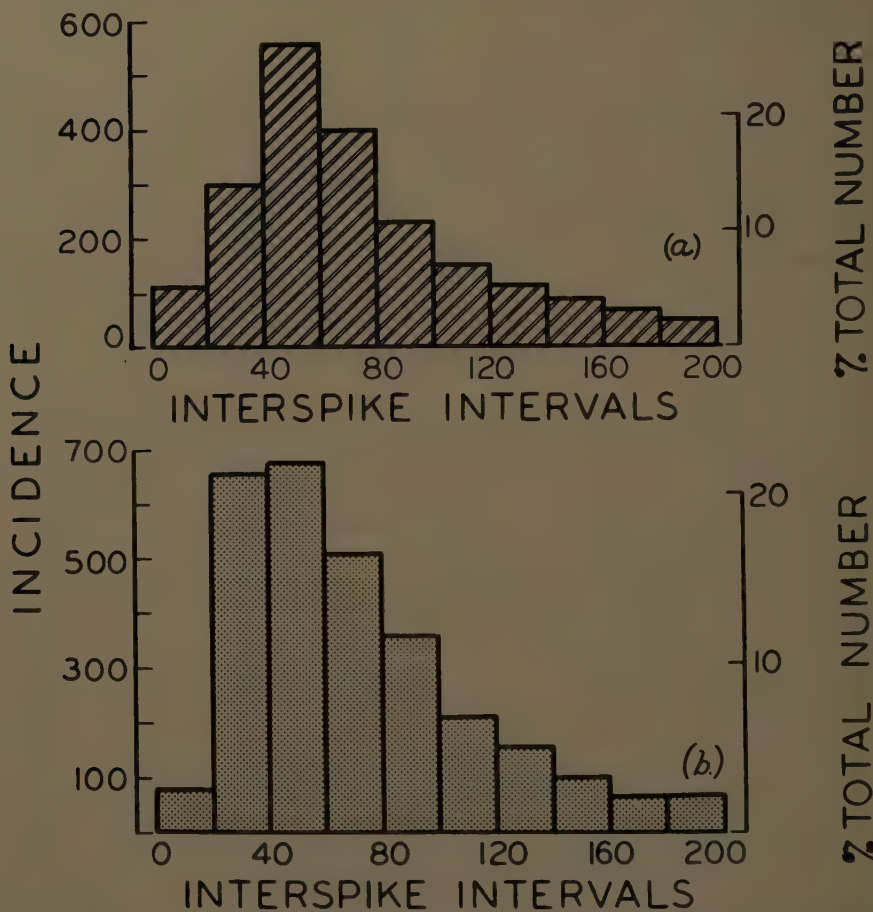

FIGURE 1. Interspike interval distributions for tape 7, run 4 (stimulation at 2-sec. intervals) and run 3 (no stimulus), unit 2L. (*a*) Stimulated run 7-4-2L: theoretical mean interval = 87.5 msec.; n = 2091 (including 92 shock artifacts); 6 per cent intervals exceed 200 msec. (*b*) Spontaneous run 7-3-2L: theoretical mean interval = 88.5 msec.; n = 3072; 6 per cent intervals exceed 200 msec.

sion respectively of the two units—see average patterns of response revealed by traveling gate in FIGURE 7, runs 7-4-1 and 7-4-2L).

Coincidences can be counted between spontaneous discharges of two neurons that are recorded by one or by two microelectrodes. The distance between the somata of two cells recorded with one electrode is likely to be small, but the exact dimension is unknown. Mountcastle *et al.* (1957) suggest that an

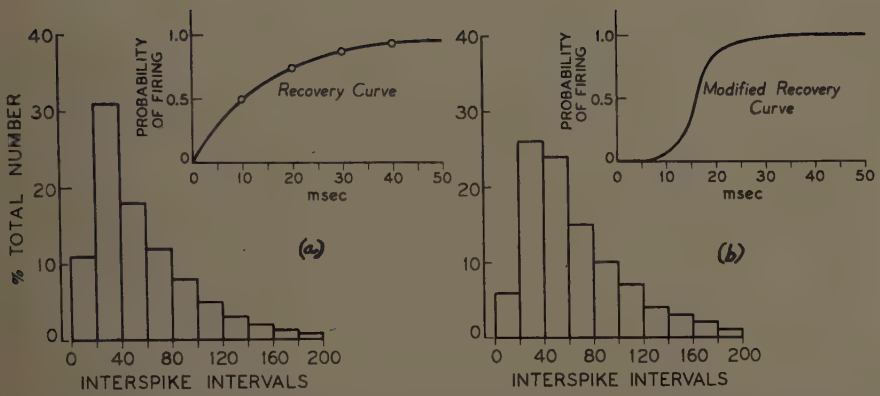

FIGURE 2. Theoretical interspike interval distributions, using two assumed recovery curves. Histograms were calculated from a Poisson distribution of spikes modified by a multiplicative factor (labeled "Probability of Firing") obtained from the indicated recovery curve as a function of the time since the last spike.

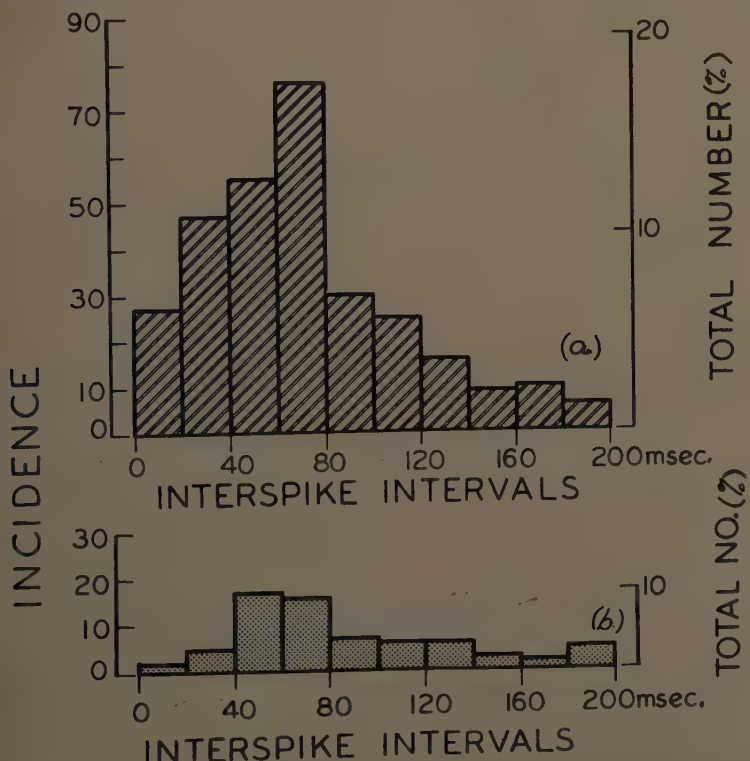

FIGURE 3. Interspike interval distributions for tape 7, run 4 (stimulation at 2-sec. intervals) and run 3 (no stimulus), unit 1. (*a*) Stimulated run 7-4-1: theoretical mean interval = 440 msec.; n = 433 (excluding shock artifacts); 69.5 per cent intervals within 0-200 msec. range; 6.5 per cent intervals within 200–300 msec. range; 4.5 per cent intervals within 300–400 msec. range; 2.5 per cent intervals within 400–500 msec. range; 17.0 per cent intervals exceed 500 msec. (*b*) Spontaneous run 7-3-1: theoretical mean interval = 1.6 sec.; n = 173; 40 per cent intervals within 0–200 msec. range; 8 per cent intervals within 200–300 msec. range; 5 per cent intervals within 300–400 msec. range; 4 per cent intervals within 400–500 msec. range; 43 per cent intervals exceed 500 msec.

initially negative unit has a minimum field of about 100 μ. By contrast, the positive-negative spike is recorded only when the electrode tip is close to the membrane (Amassian, 1953; Rose and Mountcastle, 1954). When two positive-negative spikes are recorded by a given microelectrode, the somata are presumably no more than a few microns apart. Two coincidence plots for spontaneous activity of two neurons that lie close to one another (run 7-3-2) or that are contiguous (run 7-2-1) are shown in FIGURE 4. In both pairs of

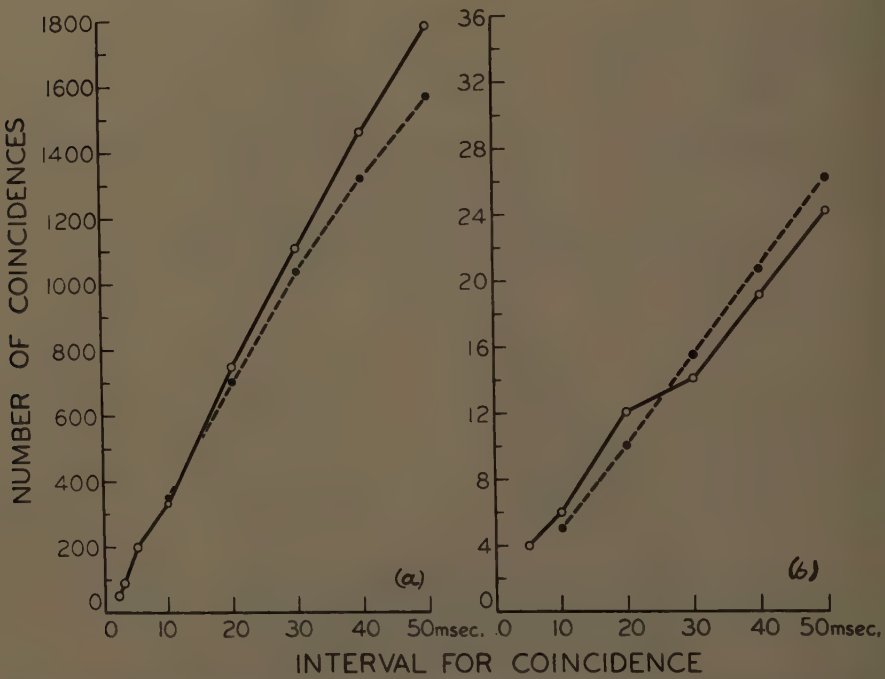

FIGURE 4. Coincidences as a function of coincidence interval for tape 7, run 3, units 2L and 2S, and for tape 7, run 2, units 1L and 1S. (Curvature of the "expected-random-coincidences" line results if gate duration exceeds minimum interspike intervals.) Key: ○——○ observed coincidences; ●- - -● expected random coincidences. (*a*) Two units recorded at one electrode position: both (−) spikes; run 7-3-2. (*b*) Two units recorded at one electrode position: both (+ −) spikes; run 7-2-1.

neurons, the increase in number of coincidences is nearly linear for coincidence intervals up to 50 msec. The expected number of coincidences, calculated on the basis of a uniform random distribution of the spikes and an independent relationship between neurons, does not significantly differ from the observed number of coincidences. We can conclude from this that spontaneous discharges of the two pairs of neurons illustrated were independent; this statement, however, must be qualified. Above an interval of 10 msec., coincidences were sought only by increasing the interval in steps of 10 msec. Such time slices may average out an excitatory-inhibitory relationship. (For example, when both the largest Group I fibers and smaller fibers of a muscle nerve are stimulated, motoneurons supplying a synergist are monosynaptically facilitated

and 0.5 msec. later are disynaptically inhibited [Lloyd, 1952].) For intervals less than 5 msec., the two pairs of units illustrated showed no greater coincidences than would be expected between independent discharges whether such coincidences were detected by the coincidence gate or from the oscilloscopic trace. The latter test revealed 32 coincidences within 0.8 msec. of the initial limb of the large spike (*a*). Such observations do not exclude the possibility of a delayed excitatory-inhibitory relationship. However, it is of interest that an excitatory-inhibitory sequence in the response of a given unit to peripheral stimulation can be demonstrated even when using a traveling 10-msec. gate. It seems unlikely in a complex polysynaptic network such as the reticular

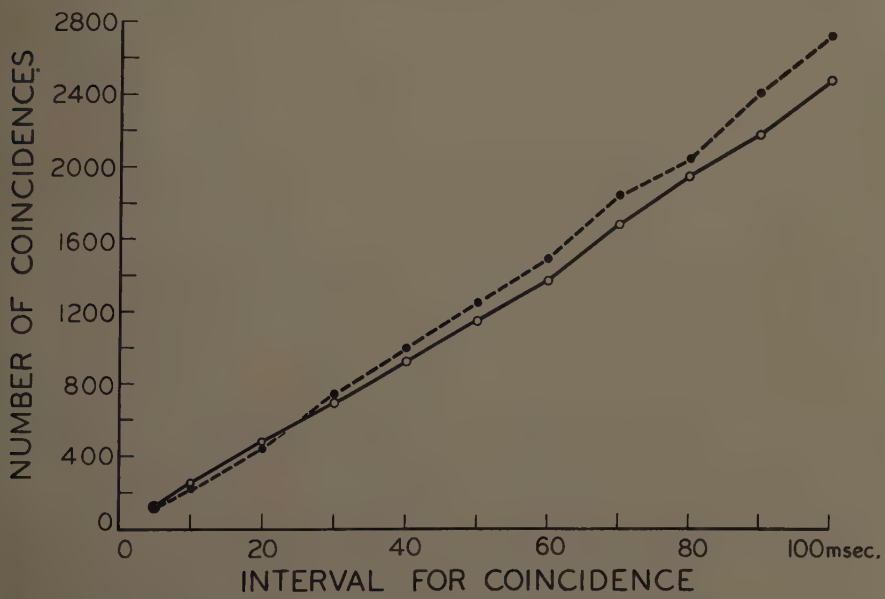

FIGURE 5. Coincidences as a function of coincidence interval for tape 7, run 11, units 1 and 2, recorded with different microelectrodes. Run 7-11: ○——○ observed coincidences; ●- - -● expected random coincidences.

formation that the transition from excitation to inhibition would occur as precisely as Lloyd (1952) showed for the motoneuron synapse.

The coincidence plots for the units illustrated in FIGURE 5 are derived from midbrain reticular neurons that are about 1 to 1½ mm. apart. The neurons appear to be independent in their spontaneous activity.

We have sought long-term relationships between reticular neurons by comparing average number of discharges of the two units during successive 10- or 5-sec. periods. An example is shown in FIGURE 6. Between periods 35 to 53, the spike count in each 5-sec. period usually changed in the same direction. If indeed the two neurons had been studied only during this period, one might have inferred that they were functionally related. However, the units showed little relationship earlier in the run. The similarity in behavior during part

of the run might be due to the chance locking in of two rhythms with differing periodicities. Alternatively, a functional relationship between the two units may have been temporarily established. The need for making observations over an extended period of time before concluding that two neurons are functionally related is too obvious to require further emphasis.

Effects of Stimulation

Brief electrical stimuli were delivered to a paw at the rate of once every 2 sec. through bipolar needles inserted into the skin. One effect of stimulation is to alter the distribution of interspike intervals (FIGURES 1*a* and 3*a*). A much more informative type of analysis is to count the number of discharges by a given neuron occurring during a gate of fixed duration that is opened by a pulse synchronized with each stimulus in the run. Spike counts are subse-

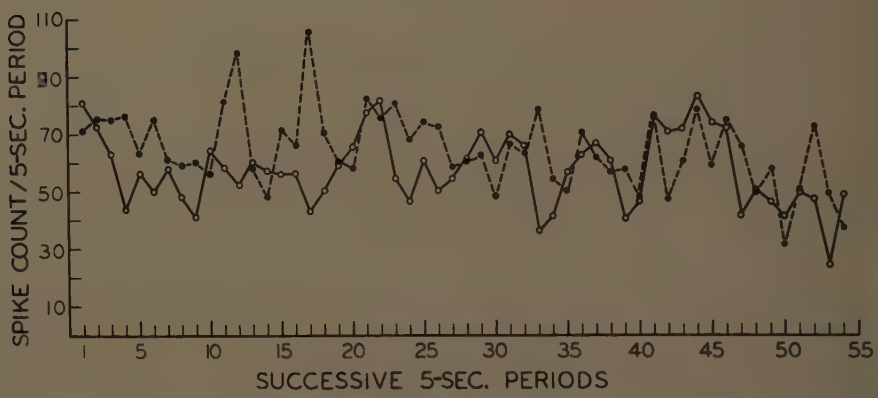

FIGURE 6. Variations in firing rate for two initially negative units recorded with one microelectrode; tape 7, run 3, units 2L and 2S. No stimulus (run 7-3-2).

quently determined when the opening of the gate is delayed by successive increments equal to the fixed duration of the gate. Data obtained by this technique from 3 simultaneously recorded neurons are shown in FIGURE 7. On the ordinate is plotted, for each gate delay, the value of the function

$$\frac{O - E_s}{E_s}, \quad \text{where} \quad E_s = \frac{D_g}{P} \cdot \frac{D_e}{D_s} \cdot N, \quad \text{and}$$

O is the cumulative number of discharges by a neuron during the 10-msec. gate; E_s is the expected number of spontaneous discharges; D_g is the duration of each gate; N is the total number of spontaneous discharges during a stimulus free run; D_e is the duration of the spontaneous run; D_s is the duration of the stimulus run; P is the interval between stimuli.

The delays for closure of the gates are shown on the abscissa. Throughout the run, a fixed 2-msec. delay was inserted between the stimulus artifact and the indicated times. This is insignificant as a source of error and is disregarded except when very early responses are discussed. It should be noted

that E_s is calculated on the basis that spontaneous discharges are uniformly distributed during a run in which no stimuli are given. This appears to contradict the statements made earlier concerning the distribution of intervals during a spontaneous run, but the value obtained from assuming a uniform distribution is of course the same as the expected value for a 10-msec. period from the observed distribution. The E_s is a convenient abstraction that facilitates comparison of the behavior of different neurons. It should be noted that the use of the expression $(O - E_s)/E_s$ leads to an apparent lack of prominence of inhibitory effects, since the expression assumes a limiting value of -1 when inhibition is complete. The effect of stimulating the ipsilateral and contralateral forepaws are analyzed in runs 7-4 and 7-6. The effect of the

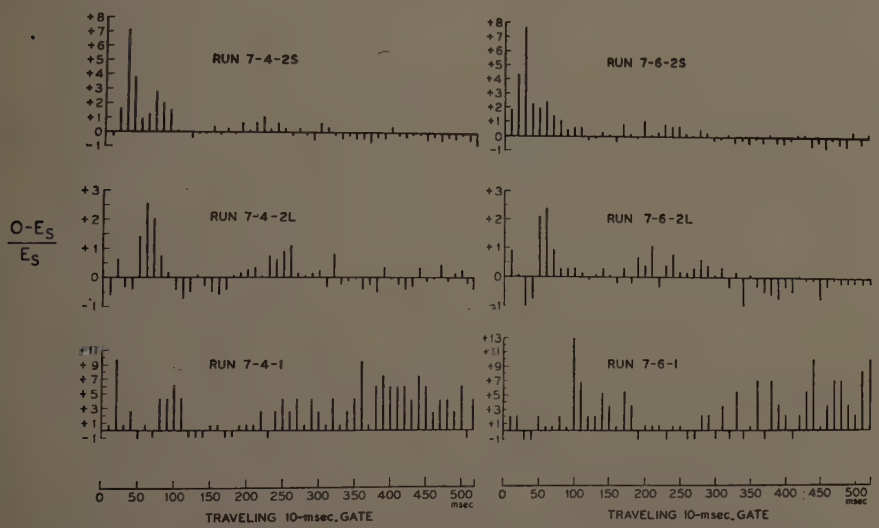

FIGURE 7. Firing patterns for three units when the site of stimulation is changed. Left column obtained with ipsilateral forepaw stimulation, right column with contralateral stimulation. Top and middle rows were obtained from two units recorded with the same microelectrode, bottom row from a unit simultaneously recorded with another electrode.

stimulus on the unit in runs 7-4-1 and 7-6-1 persists for at least 520 msec. after the stimulus. Furthermore, the behavior of the neuron is not uniform throughout the period of 520 msec. following the stimulus. This unit shows a phase of late response to stimulation of either paw, but the pattern of early activity differs according to the site of stimulation. The upper rows show data from two units recorded by the same microelectrode. The responses of both units show cyclical phases of facilitation and depression. The responses in runs 7-4-2L and 7-6-2L peak at about 60 msec. and again at 200 to 240 msec. Peak response occurs earlier in the top row of plots.

Responses of another pair of neurons to stimulation of the contralateral and ipsilateral forepaws, respectively, are analyzed in FIGURE 8. The predominant effect of stimulating the contralateral forepaw is either partial or complete inhibition of spontaneous discharge of both neurons for a period of at least

420 msec. after the stimulus. Moreover, the total number of discharges occurring during the 420-msec. period after the stimulus (O) is considerably less than the number (E) calculated on the basis that the discharges occurring during the stimulus run are uniformly distributed during the period between stimuli (compare O/E in runs 7-9-1 and 7-9-2). The behavior of the unit in run 7-10-2, following stimulation of the ipsilateral forepaw, shows a complex sequence of inhibition-excitation-inhibition for the period up to 420 msec. after the stimulus. We are unable to determine if the other unit (top row) was excited or inhibited when the ipsilateral forepaw was stimulated (run 7-10-1).

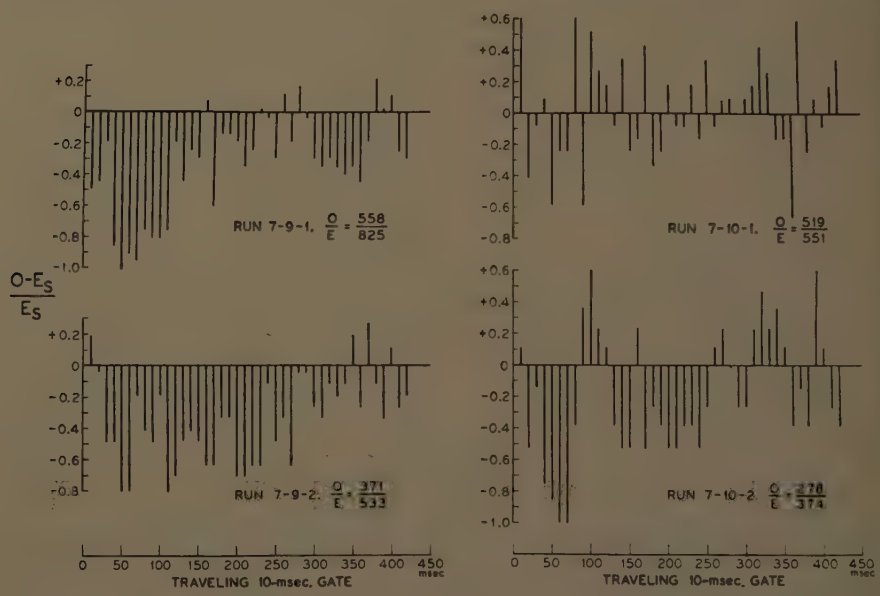

FIGURE 8. Firing patterns for two units recorded with two microelectrodes, when the site of stimulation is changed. Right column is obtained with ipsilateral stimulation, left column with contralateral stimulation.

It is apparent that the above method of analysis does not discriminate between repetitive responses of the neuron on a few occasions and single responses on more numerous occasions, during a specified 10-msec. period after the stimulus. Nevertheless, reticular neurons often show repetitive discharge at high frequency (Amassian and Waller, 1958). Such repetitive discharges can be identified by counting only the first spike to occur in each gate period. This total is compared with the total obtained by counting all the spikes during the gate period; this check was performed for two units, and the difference in totals clearly indicated the presence of such repetitive discharges by one unit.

When responses are evoked from a pair of neurons within a given period after the stimulus, the number of coincidences occurring in this period is obviously greater than the number of random coincidences occurring in a period

of equal duration during a spontaneous run. While an increase (or decrease) in the number of coincidences in a specified period may be of great importance in the analysis of incoming signals by the nervous system, such observations do not necessarily indicate a functional relationship between the neurons. Coincidences during stimulus runs were counted for three different pairs of neurons, but only one of the pairs, recorded by a single microelectrode, yielded data from which conclusions can be drawn (see TABLE 1). Only a single discharge per gate was permitted to reach the counters. Discharges of neuron S were separately counted on one channel by electronically canceling the larger spike of the L neuron. The probability of discharge (P_S or P_L) of neuron S or L for a given gate delay was estimated by dividing the cumulative count during the gate by the number of stimuli ($n = 93$). The actual probability of coincidence was estimated by dividing the coincidence count by the number

TABLE 1
COINCIDENCE OF RESPONSES TO STIMULUS
Run 7-4-2: Two Units Recorded at Same Position of Electrode

Traveling 10-msec. gate	P_S	P_L	Calculated $P_S \times P_L$	Observed P of coincidence
Start 10 msec.	0.27	0.11	0.03	0.05
20	0.78	0.09	0.07	0.08
30	0.58	0.64	0.04	0.03
40	0.22	0.22	0.05	0.10
50	0.27	0.42	0.11	0.09
60	0.42	0.34	0.14	0.13
70	0.35	0.19	0.07	0.06
80	0.35	0.12	0.04	0.05
90	0.15	0.05	0.01	0.01
S gate 20, L gate 50	0.77	0.46	0.35	0.37

of stimuli. If the discharge of the two neurons is independent, the product $P_S \cdot P_L$ gives the probability of random coincidence. Except in one instance, the actual probability of coincidence agrees well with the calculated probability of random coincidence. The agreement also holds when the 10-msec. gates for the S and L units are started 20 and 50 msec., respectively, after the stimuli (these times were chosen because the peak responses of the two neurons occur prior to the closure of these gates).

Discussion

Our preliminary analysis of the effect of a peripheral stimulus on midbrain reticular neurons indicates that the spontaneous pattern of discharge is perturbed for a considerable time after the stimulus. The changes may be subtle and may be missed by visual inspection of the spikes. The effect of a stimulus is nonuniform during the period between stimuli. Cyclical facilitation and depression of responses is revealed by using a computer for the analysis of the data. A spontaneously active neuron cannot be said to be either *excited* or *depressed* following stimulation. Instead, the effect of stimulation

should be specified for a series of times after the stimulus. Two neurons recorded simultaneously do not respond in an identical manner to a given stimulus. Peak responses may occur at different times after the stimulus regardless of whether the neurons are recorded by a single electrode, or whether they are recorded by two electrodes situated 1 to 2 mm. apart.

The distribution of interspike intervals during spontaneous runs has already been discussed. The distribution is complex and cannot be fitted by a simple independent random model. Our conclusions agree with those derived by Kuffler *et al.*, (1957) from a study of spontaneous discharge of retinal ganglion cells.

Failure to detect coincidences between spontaneous discharges of neurons that lie close to one another, other than those expected on a random basis, requires comment. Our technique of analysis may be too crude to reveal subtle relationships between neurons. Discharge of a reticular neuron presumably requires bombardment by impulses in more than one presynaptic fiber. The number of active fibers is probably a small fraction of the total number of presynaptic fibers related to the neuron. The variability in behavior of single reticular neurons implies that the set of active presynaptic fibers varies within the population of fibers impinging on a given reticular neuron. Such factors would be expected to reduce the likelihood of detecting a significant relationship between discharges of two neurons picked at random by a single microelectrode. It was previously inferred from the differences in behavior of neurons that lie close to one another that excitation of the reticular net was not transmitted by sequential spread between adjacent neurons (Amassian and DeVito, 1954). If significant anatomical connections between reticular neurons occur at some distance from the somata (Scheibel and Scheibel, 1958), the chances of recording related activity in two neurons is still further diminished.

Summary

The activities of simultaneously recorded midbrain reticular neurons are analyzed with a computer. A comparison is made between spontaneous and evoked activity in neurons that lie close to one another and between neurons that are separated by 1 to 2 mm.

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